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ABSTRACT

This volume is the twelfth in a series of 29 coordinated MINNEMAST units in mathematics and science for kindergarten and the primary grades. Intended for use by first-grade teachers, this unit guide provides a summary and overview of the unit, a list of materials needed, and descriptions of four groups of lessons. The purposes and procedures for each activity are discussed. Examples of questions and disucssion topics are given, and in several cases ditto masters, stories for reading aloud, and other instructional materials are included in the book. This unit is concerned with measurement of length, area, volume, and time durations. After reviewing the comparison of objects in a previous unit, the idea of standard units is introduced; a variety of tools (e.g., paper clip chains, popsicle sticks, rulers) is used in measurement activities. The pendulum and a variety of clocks are used in activities related to time. (SD)

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MINNEMAST

MEASUREMENT WITH REFERENCE UNITS

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UNIT 12

MINNEMAST

2

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

MINNEMAST COORDINATED MATHEMATICS–SCIENCE SERIES

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	2.	CURVES AND SHAPES
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	5.	INTRODUCING MEASUREMENT
	6.	NUMERATION
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	8.	OBSERVING PROPERTIES
FIRST GRADE	9.	NUMBERS AND COUNTING
	10.	DESCRIBING LOCATIONS
	11.	INTRODUCING ADDITION AND SUBTRACTION
	12.	MEASUREMENT WITH REFERENCE UNITS
	13.	INTERPRETATIONS OF ADDITION AND SUBTRACTION
	14.	EXPLORING SYMMETRICAL PATTERNS
SECOND GRADE	15.	INVESTIGATING SYSTEMS
	16.	NUMBERS AND MEASURING
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	18.	SCALING AND REPRESENTATION
	19.	COMPARING CHANGES
	20.	USING LARGER NUMBERS
	21.	ANGLES AND SPACE
	22.	PARTS AND PIECES
THIRD GRADE	23.	CONDITIONS AFFECTING LIFE
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UNIT 12

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

4

MINNEMAST

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MEASUREMENT
WITH
REFERENCE UNITS

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Suggested Teaching Schedule for MINNEMAST First Grade Units

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Complete List of Materials for Unit 12

(Amounts based on a class of 30)

total number required to teach unit	item	lesson in which item is used
35	**Student Manuals	
1	flannel board and set of flannel strips of various lengths	1
30	craft sticks, 1 per child	2
600	(#1) paper clips, 20 per child	3,4
30	strips (11" x 1-1/2") of heavy white paper, 1 per child	3
30	*rulers with inches and centimeters indicated, 1 per child	5,6,10
9	*pegboards	6,11
30	*golf tees, 1 per child	6
1	***set of property blocks (optional)	7,8
30	short (10" to 12") strings or strips of paper, 1 per child	7
	cards illustrating simple open curves (optional)	7
	commercial puzzles requiring a comparison or matching of areas (optional)	8
	many identical objects such as lima beans, buttons, paper clips, small blocks or counters for measuring areas	9
450	*counters (1" x 1") 15 per child	10,12
400	*golf tees, 50 same color per group of 4	11
8	*pieces of plastic clothesline, 24" long, 1 per group of 4	11
8	pieces of masking tape, 2" long, 1 per group of 4	11
2	containers of slightly difference volume, but markedly different shape, such as a 10- to 12-oz. soup can and an 8-oz. cottage cheese container	13

	sand or salt to fill the two containers	13
30	frozen juice cans (6-oz.), 1 per child, brought by the children	13
30	Jell-O boxes (3-oz.), 1 per child, brought by the children	13
1800	* corks (#4 size), about 60 per child or enough to fill a 12-oz. container	13
	* plastic containers as follows: 40 (1-oz.); 4 (8-oz.); 20 (16-oz.) pints; 16 (12-oz.) tall; 16 (12-oz.) shallow bowls; 2 (32-oz.) quarts; and 1 (64-oz.) half-gallon	14,15,16
7 or 8	milk cartons (1 qt.), 1 per group of 4, to be brought by the children	14
1	large container of water	14
1	sink or large tray	14,15,16
10	trays, 1 per group of 4	14
30 ounces	* plasticine	16
	thin plastic such as Saran Wrap	16
30	* order cards (>) from number card set, 1 per child	18,19
30	duration cards, 1 per child	18,19,20
several	winding toys such as music boxes, to be brought by the children	21
1	* pendulum, 5 or 6 feet long	22,23
1	wall clock with second hand	24
11	* pendulums, lengths specified in the lesson	24
1	* demonstration clock face with minute and hour hands	26
30	* small cardboard clock faces with movable hands	26,27,28

* kit items as well as

** printed materials available from Minnemath Center, 720 Washington Avenue S.E., Mpls., Minnesota 55455

*** available from The Judy Company, 310 North Second Street, Minneapolis, Minnesota 55401

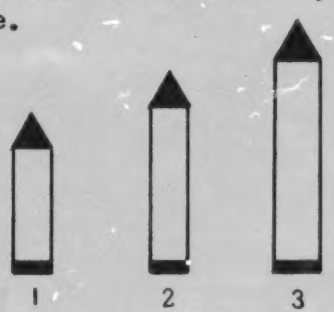
INTRODUCTION

Measurement is one of the most important operations of science. It usually includes the process of assigning numbers to various properties of objects or events, so that we can indirectly compare those properties by comparing the numbers without bringing the objects together for direct comparison. To compare the property of length, we might ask, "Which is longer -- the teacher's desk or the principal's?" We need not bring the desks together, nor rely on our memories of how they look. Instead, we can ask, "Which desk has had the greater number of units assigned to its length by a measuring process?"

Obviously the measurements must be made in the same way, if we are to be able to make comparisons. We must use the same standard units. The number 39 is greater than the number 4, but 39 inches in not greater than 4 feet. And we should not compare the length of the teacher's desk with the width of the principal's, if we want to know which desk is longer.

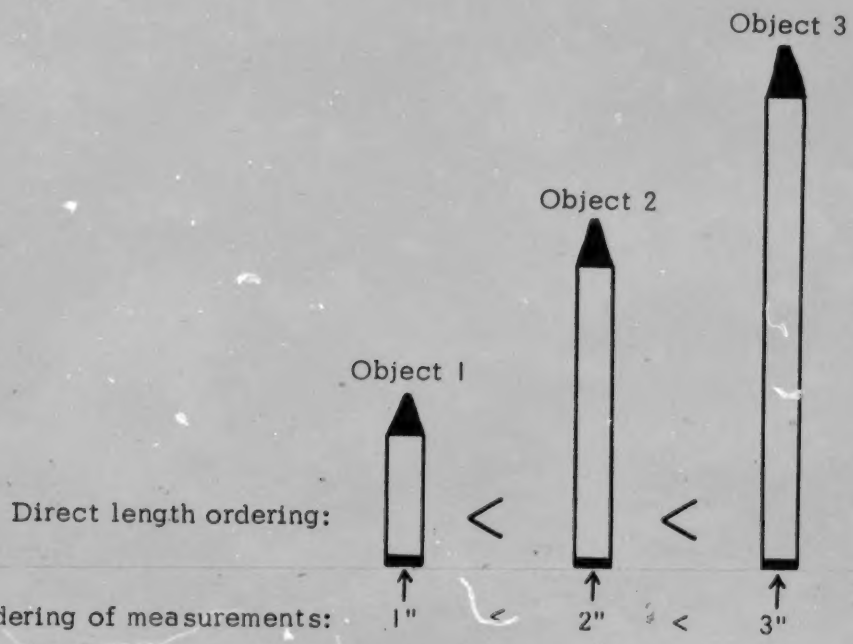
We can assign numbers to any kind of property or event that we can describe precisely, provided we can specify a measuring process -- a way of making the assignment. We can even measure properties of "things" that are not objects, for example, time duration.

There is a basic kind of measurement in which we compare properties of two objects directly, repeating such comparisons until we have ordered or ranked the objects from least to greatest in magnitude.



No numbers need be involved in this; our ordering just ranks them, according, say, to length. Such a ranking has many of the properties of numerical ordering. It is transitive: if object 2 is longer than object 1, and object 3 is longer than 2, then object 3 is longer than 1. We need not compare objects 3 and 1 directly.

If we order the numerical measurements of objects 1, 2 and 3, we shall find the same relationships. If the number assigned to object 2 is greater than the number assigned to object 1, and the number assigned to object 3 is greater than that assigned to object 2, then the number assigned to object 3 is greater than that assigned to object 1.



Without using numbers assigned by measurement, the children have done a good deal of work comparing and ordering objects according to the magnitude or size of some common property. Direct comparisons, comparisons with reference units, and various orderings have been made for lengths, areas, volumes, and time-durations. If most of the children in your class never studied Unit 5, or if they need review, you might wish to use activities from Lessons 1, 8, 12 and 16 of Kindergarten Unit 5, Introducing Measurement.

In this unit, we will be teaching the children several processes for assigning numbers to properties of objects and durations of events. For classroom work, measurable properties should be those the children can readily identify, and which do not change too rapidly for them to work with. (Sometimes when we want to

see how an object has changed, we can describe the change as the numerical difference between two measurements made on the object at different times.)

The lessons are designed to bring out several ideas about the choice of units for measurement. It is important, for making comparisons and for communicating with others, to have standard units of measurement that are as nearly alike as possible and that everyone uses. The standard unit chosen is arbitrary. To demonstrate this, paper clips and handspans are used at first. But the initial determination of the magnitude of the familiar inch, centimeter, foot, etc. was equally arbitrary.

The main considerations in choosing measurement units are convenience in use and in communication. When the length of a king's arm was chosen as a measuring unit, it would have been impossible for the king to be present whenever it was desirable to compare the length of some object with the length of his arm. Therefore replicas of that length unit were made and the unit became the basis for what we now call a "yard." While a handspan is convenient for approximations, people's hands differ in size and handspan length changes as the owner of the hand grows. It even changes from time to time, according to how one spreads his fingers. Handspans are not accurate, even if we do not care much about communicating our measurements. Measurements reported in paperclip lengths may be convenient, but they will provide accurate comparisons only for those who use paper clips of just the same size. Thus people have agreed on certain arbitrary units, such as inches, centimeters, ounces, grams, seconds, and so forth. The process of measuring has been standardized, so that measurements made by everyone will be communicable and comparable.

The importance of using a standard unit (such as the inch or centimeter) which has been commonly agreed upon is so obvious that we tend to assume that this approach is known and understood even by young children. It is true that at an early age children may accept and use the common measuring standards or scales of their culture, but a child may understand neither the arbitrariness of the choice nor the principle of

defining a common standard. That is why children begin their measurements, in these lessons, by using paper-clip chains to measure length, corks to measure volume, and pendulums to measure time durations. Then, in later activities, they use common standard units such as inches, centimeters, quarts, seconds, etc.

The children measure and record values of length, area, volume and time duration, and determine time order. To record their results they use symbols for comparisons ($>$, $<$, $=$). Other activities include: counting and addition games making distinctions among perimeters, areas, and volumes, and clock and calendar drills. The children learn to use centimeter and inch grid sheets for their area measurements and they develop "clocks" which provide time-measurement standards.

To simplify the recording of the results of measurements, it is convenient to express the measurements and their relationships by means of symbols, so the children will use symbols a great deal. Objects will often be symbolized by letters, i.e., <u>length of A</u> means "the length of object A." Inequalities will be represented by the symbols $>$ and $<$ meaning respectively "is greater than" and "is less than."

If the volume of object M is less than the volume of object N, we may write:

$$\text{Volume M} < \text{Volume N}$$

This is sometimes abbreviated further as:

$$V_M < V_N$$

(Note that the <u>larger open end</u> of the "greater than" symbol is always toward the <u>larger quantity</u>; the <u>small pointed end</u> is toward the <u>lesser</u> quantity. This is a memory aid to which you should call the children's attention.)

Often it is impossible to decide by measurement which of two quantities is the larger, or, for the immediate purpose, the quantities are close enough to each other so it doesn't

matter. We then say that the two quantities "appear to be the same" or that "by our measurement they seem to be equal." This relationship we will symbolize by $\doteq$. For example: "Area S $\doteq$ Area T" means "the area of S appears to be the same as the area of T."

"Duration X $\doteq$ 5 sec." means "the duration of event X is about 5 seconds." The symbol $\doteq$ may also be read as: "is approximately equal to," "is measured to be the same as," or "is nearly the same as."

Even though our most careful measurements cannot determine any difference between two quantities, experience tells us that we may find a difference using more precise methods or instruments (microscopes, for instance). Therefore, the results of comparisons should not be expressed as "equal" (=).

The equal sign (=) should be used only in two cases:

1. Between different names for the same thing (3 + 2 = 5).

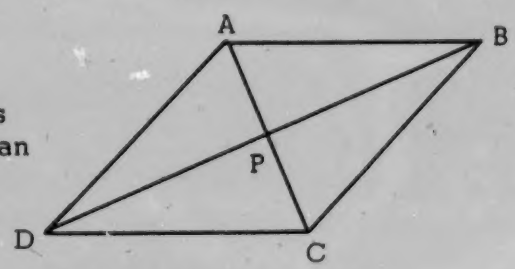
   It may be determined by counting that two quantities are equal. If two sets are equivalent, $N_A = N_B$. This means that the number of members of Set A is equal to the number of members of Set B. For example: "The number of apples in this bag equals the number of oranges on the table."

2. In definitions and logical deductions.

   "All the sides of a rhombus are equal." (Definition)

   AB = BC = CD = DA

   "The diagonals of a rhombus form right angles." (This can be proved logically.)



Angle APB = a right angle.

NOTES ON TEACHING THE UNIT

The material in these lessons could be covered more quickly and easily if you took a dominant role, immediately providing information and answers to student questions. It is desirable, however, for the children to discover the need for agreeing upon and adopting a standard unit of measurement. This need will be developed by having the children carry out a variety of measurements, some of which seem very inefficient. The specific amount of direction or information children will need depends upon their maturity, ability and prior experience. This can be determined only by you.

After standard measurement units have been adopted by the class, several activities are suggested to develop the children's skill in making measurements and recording the results.

One can expect the results of a measurement to vary and you should accept a slight variation in a repeated measurement. This is not taught explicitly but be on the lookout for such cases. Suppose several children measure the same object and get differing results, or one child repeats a measurement and his results vary. If there is no one choice obviously more reliable than the others, do the children select one of the results to be the measurement? Or do they realize that measurements are approximate and accept some variation? Recording measurement data in histogram form will often show that, when results vary, a "most likely" value can be selected and used as the "accepted" value.

Children may raise questions as to why we use different systems of units for measuring the same quantities -- inches and centimeters, for instance. It is done for several reasons:

1. It is desirable for the children to be familiar with more than one system, thus emphasizing the arbitrariness of the choice of units.

2. It appears certain that the metric system will become increasingly used in this country.

3. As their knowledge of the metric system develops, the

children will appreciate another application of the base-ten notation.

4. Some basic ideas of accuracy will be established. Measuring the length of a room in centimeters will yield larger numbers than measuring it to the nearest inch. Measuring to the nearest centimeter is more accurate than to the nearest inch. Two objects which appear to be the same length when measured to the nearest inch may be seen to differ in length when measured to the nearest centimeter.

This unit is composed of four sections, each with a list of purposes and a commentary:

Section 1: Measuring Length
Section 2: Measuring Area
Section 3: Measuring Volume
Section 4: Measuring and Ordering Time Durations

Each section contains several lessons for which the needed materials and procedures are described.

Although a lesson is usually best taught in one class period, you may be able to combine several for your class or you may find that more than one period is needed for a particular lesson. For example, all of Section I might take eight or nine days.

OBJECTIVES

A child who has completed this unit should:

- Be able to make simple comparisons of length, area, volume and time duration.
- When confronted with two objects which cannot be directly compared, search for some reference or standard unit and use it.
- If common standards such as inches are not available, be able to use other objects (e.g., erasers) as reference units to make a measurement.

- Be able to make an approximate determination of length, area, volume and time duration using standard measuring devices.

- Be able to express and record the results of a measurement.

- Be able to interpret recorded results of measurement, either orally or by using them in some correct way.

SECTION I MEASURING LENGTH

PURPOSE

In this section a child:

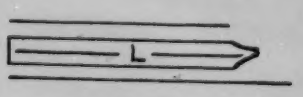
- Sees the need for standard units of measurement.
- Discovers that standards are chosen arbitrarily.
- Demonstrates his ability to measure the length of various objects.
- Practices recording length measurements and communicating the results.
- Uses some common standards for measuring length.

COMMENTARY

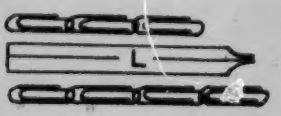
Some concepts and techniques involved in measuring length are explored in this section. The children make a measurement comparing an object with two reference objects which bracket it (one greater and one less in length). Their attention is centered on what is involved in choosing a standard of measurement rather than on the rote use of a foot ruler or meter stick. Therefore, the first unit of length used is purposely not the inch or the centimeter. After the children learn how to measure length, they use the process as a means for describing and classifying various objects. Near the end of this section, the ruler is introduced as a standard measuring device.

Quantitative comparison is developed as follows:

- A length is measured and found to be between two other lengths. (Lessons 1, 2 and 3.)

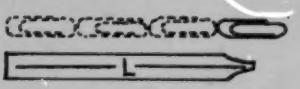


- The reference objects are duplicated to obtain a standard reference set, consisting of multiples of one unit, e.g., two chains of paper clips. The length is found to be longer than one chain of units but shorter than another. (Lesson 3, Activities A and B.)



- The same result is found using only a chain longer than the length. (Lesson 3, Activities B and C.)

- One reference unit is used repeatedly. (Lessons 2 and 4.)



The lessons in this section are only a sampling of what might be done. Many similar and equally valuable activities will occur to you. The amount of time to devote to this section depends on the maturity of your class and can only be determined by you. Any or all of the activities could be used in small groups, but it is likely that you will want to develop some with the whole class, making others available for use by small groups or individuals for enrichment or reinforcement.

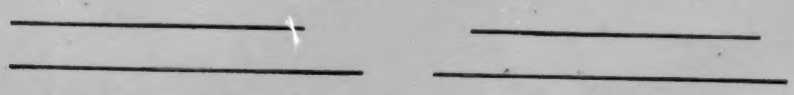
Several supplementary activities can be found following Lesson 7. They are appropriate for use as extensions of some lessons, as reviews at the end of the section, or as refresher activities later in the year.

Lesson 1: COMPARING OBJECTS ACCORDING TO LENGTH

The lesson is intended to:

- Develop each child's understanding of the concept of length of an object and his ability to compare the lengths of two objects.
- Give practice in arranging (ordering) three or more objects according to length.
- Review the transitivity principle by applying it to length relations.

The children select objects from their desks to be compared. If two objects are not side by side, the length of one may appear much greater than the length of another, when actually the lengths may be about the same. Length comparisons are best made by placing the objects side by side with one pair of ends matching or with both ends of one object extending beyond those of the other.



It is desirable but not necessary that the objects used for the comparisons in this lesson have one dimension obviously greater than their other dimensions. Objects such as books and sheets of paper may be used if the dimension along which the lengths are to be compared is specified. Ordinary usage distinguishes length, width, and the diagonal, but any of these is a length dimension.

The various activities of this lesson will obviously require considerable class discussion in order to develop new ideas. Use diagrams and the comparison symbols often in these discussions in order to familiarize the children with the advantages of this kind of recording.

If some children cannot compare and order objects by the property of length, additional work may be necessary before proceeding with these lessons. See Unit 5, Introducing Measurement, (Lessons 2, 3 and 5) for review activities.

MATERIALS

- objects from each child's desk
- flannel board
- set of objects of varied lengths for flannel board

PROCEDURE

Activity A

Have each child choose five objects of different lengths from his desk and put them out of the way at the front of the desk. Ask that each child pick up two of the objects and place the longer object at the right of the shorter one. Then ask one child to show the class how he made the comparison of his objects.

Next have each child select a third object from his set and decide which of the three belongs in the middle (because it is longer than one object, and shorter than the other).

Now ask each child to arrange his entire set of five objects in order by length, with the longest at his right. Have a few children justify their arrangements by direct, side-by-side comparisons of the objects.

Bring out a demonstration set of lengths and the flannel board. Have someone arrange the lengths in order, with the longest at the right. Ask the children if they can think of a way to record this order. Accept any reasonable recording method that the children suggest. It is possible that someone will remember previous measurement units and suggest this notation:

$$A < B < C < D < E$$

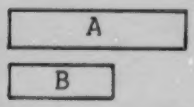
If you think it is necessary, review the meanings of the greater than (>) and less than (<) symbols. You may wish to tell the children that an easy way to keep from confusing these symbols is to remember that the arrow always points to the smaller measurement. Also review the approximation symbol ($\doteq$), which is referred to in MINNEMAST as "the appears to be the same as" symbol. Explain that we use this symbol

because we can never obtain a truly precise measurement because of our own mistakes in measuring or because of inaccuracies in our equipment.

Activity B

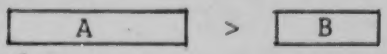
Using several flannel board objects of varied lengths, have the class carry out the following steps:

Choose two objects, line up one set of ends and decide which object is longer.



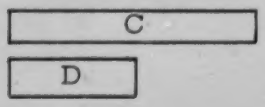
Length A > Length B

Encourage the children to describe this comparison by saying, "The length of A is greater than the length of B." Ask the children if they can think of a way to record this.



or Length of A > Length of B

Choose two other objects and compare them.

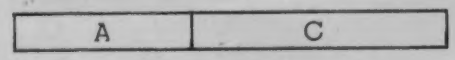


Length C > Length D

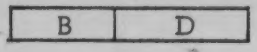
Again ask the children to record this on the board.

Length of C > Length of D

Take the longer object from each pair and place the two objects end to end.



Take the shorter objects from each pair and place the two objects end to end.



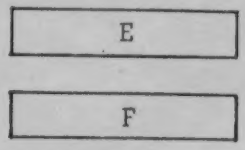
Compare the combined length of the pair of longer lengths with the combined length of the pair of shorter lengths.

By this procedure the child should discover that the length of the combination of the longer bars is greater than that of the shorter bars.

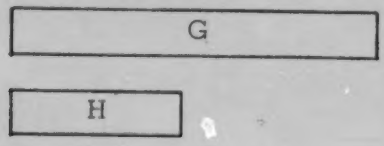
Activity C

This is a modification of Activity B. Encourage the children to record each step on the board. Have the children:

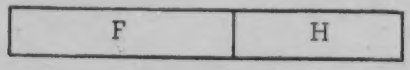
Find two objects that are about the same length.



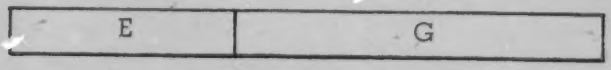
Find two objects that are not of the same length.



Put one object from the first set at the end of an object from the second set.



Put the other objects end to end.



Compare the length of the first combination (F + H) with the length of the second combination (E + G). Conclusion:

Length of F + Length of H < Length of E + Length of G.

The children should now understand that adding bars of equal length to those of unequal length does not change the inequality.

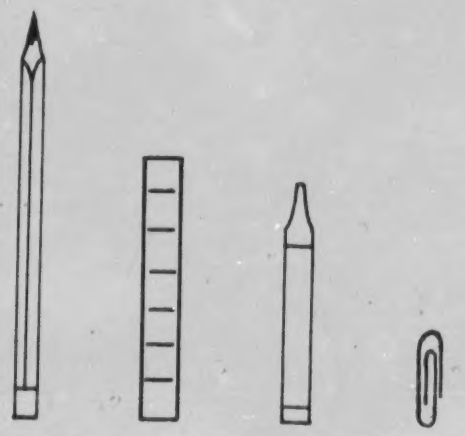
Activity D

Repeat Activities B and C, but now have the children predict the results before the combinations are formed and compared.

Activity E

To show that a prediction can be made only in certain cases (those above), have the children carry out the following activity:

Have each child select four objects of different lengths, arrange them in order from longest to shortest, and name them A, B, C and D.



IF WE PUT THE LONGEST AND THE SHORTEST OBJECTS TOGETHER, END TO END, HOW WOULD THEIR TOTAL LENGTH COMPARE WITH THAT OF THE OTHER TWO TOGETHER?

After the children have made whatever predictions they can, let them make the comparisons.

If the children have picked their own objects, the class results should include cases where:

Length of A + Length of D > Length of B + Length of C

others where:

Length of A + Length of D < Length of B + Length of C

and perhaps cases where:

Length of A + Length of D $\doteq$ Length of B + Length of C

DOES THIS COMPARISON COME OUT THE SAME FOR ALL OF YOU? (No.)

When the results have been discussed:

CAN YOU TELL WHAT THE RESULTS WILL BE BEFORE TRYING THE COMPARISON? (Not always.)

In this case, a valid conclusion is, "We can't tell how the new comparison will come out from just the original comparisons."

Lesson 2: MEASURING LENGTH WITH STANDARD UNITS

The activities of this lesson are designed to:

- Emphasize that measuring usually involves assigning numbers to properties of objects.
- Give each child experience with the repeated use of a single reference unit in making the measurement of a length.
- Establish the need for using standard units so that measurements can be compared and communicated meaningfully.
- Develop the language and notation for recording measurements.

The first two activities in this lesson make the children aware of the confusion which arises when measurements are made with a variety of reference objects. Try to have this discovery come from the children with a minimum of guidance from you.

The third activity reveals to the children the advantage of having all of them use the same reference unit — a standard. It also gives them practice in measuring by repeated placing of a single reference object.

When using any of the worksheets, always be sure to read the instructions aloud to the class as they follow along in their manuals.

MATERIALS

-- for each child --

- crayons
- standard stick (craft stick, Popsicle stick, Minnebar, etc., the same for each child)
- Worksheets 1 through 4

PROCEDURE

Activity A

The length of the classroom is to be measured, using the children as reference objects. To bring out the need for a standard unit, select smaller children as reference objects for the first measurement and larger children for a second measurement of the room. The children should not realize this distinction until after the measurements have been made.

If the floor is clean enough, have one group of children lie on the floor head to feet, so they stretch in a line the length of the room. (Standing with arms outstretched would be an alternate method.) Those who are not reference objects can count the number of children required.

The result should be written on the chalkboard:

Length of room > length of 10 children, and,

Length of room < length of 11 children

This should be read, "The length of the room is greater than the length of ten children and the length of the room is less than the length of eleven children."

If the measurement appears to come out a whole number of reference units, write on the board:

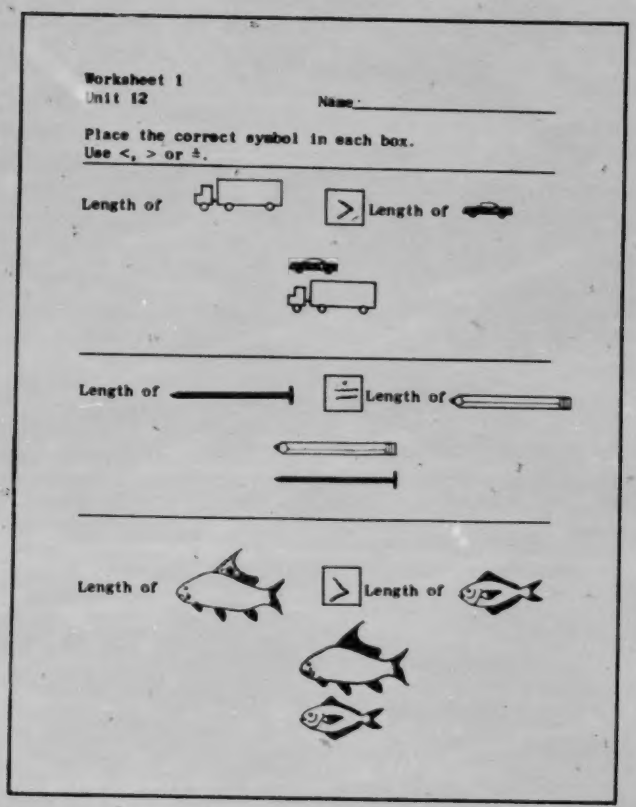
Length of room ≐ length of 10 children

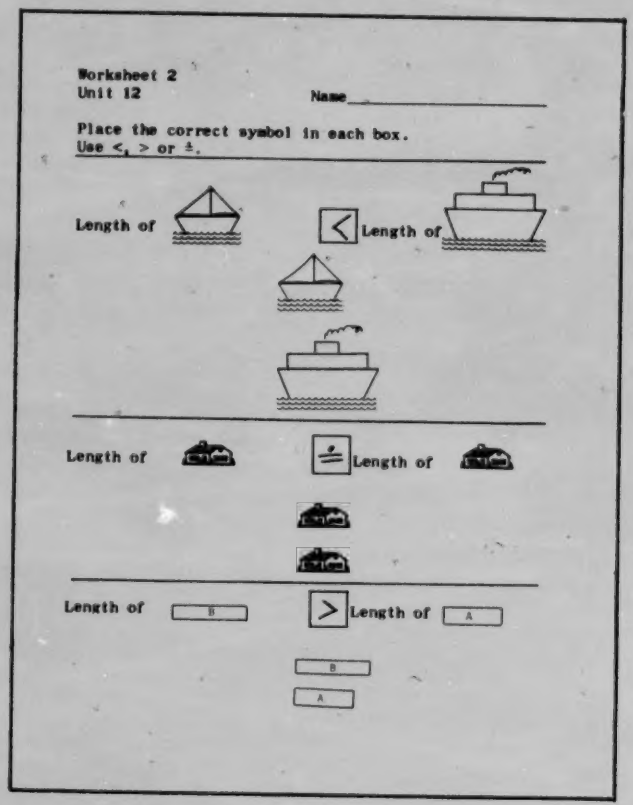
This should be read, "The length of the room appears to be the same as the length of ten children."

To check the first measurement, a second group of (taller) children should stretch in a line the length of the room. Record and verbalize the second measurement. If this results in a different number of units, a discussion about which measurement is correct and how a consistent value could be obtained should follow. This discussion will probably lead to the discovery that a standard unit for measuring length is necessary.

Activity B

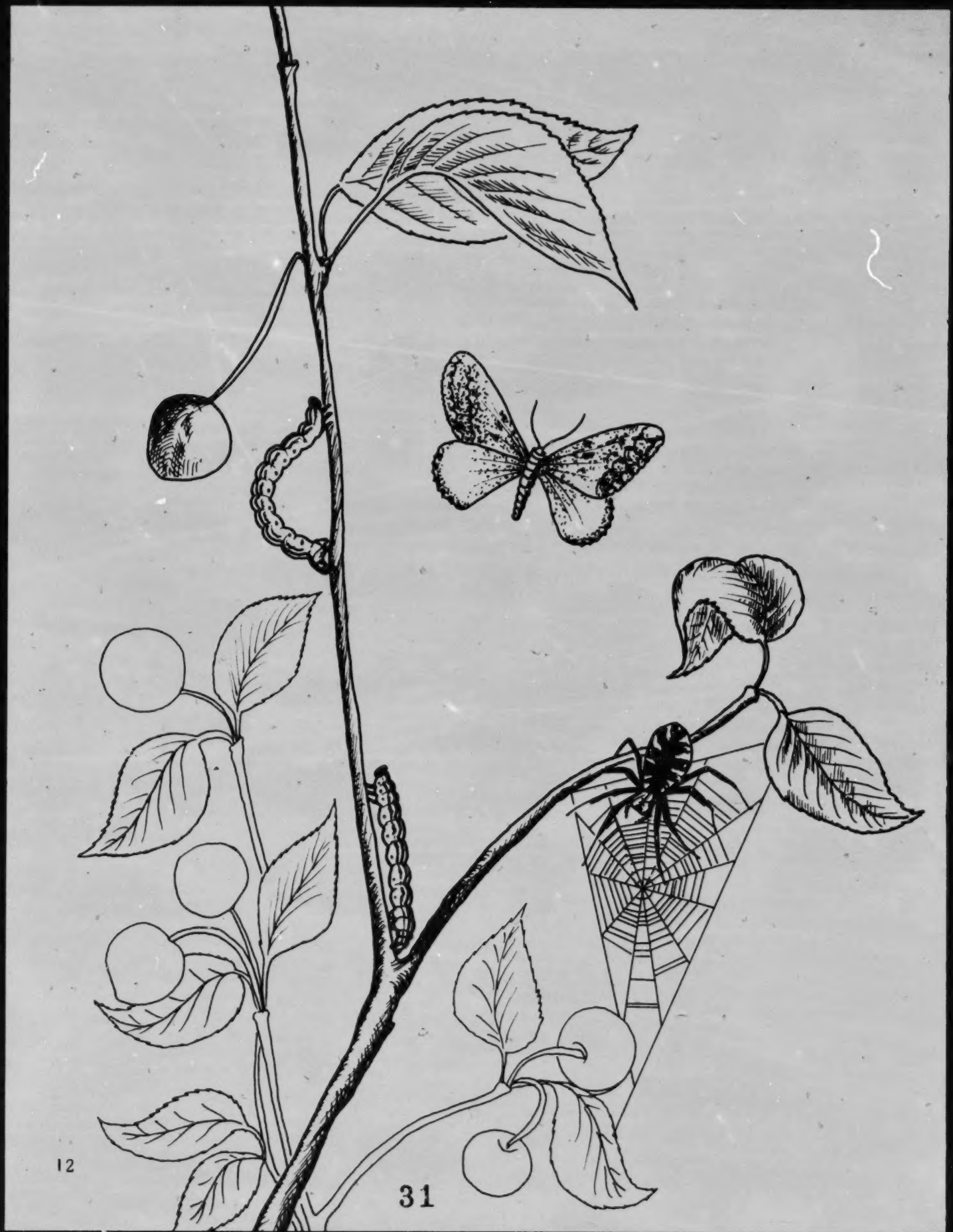
Distribute the Student Manuals and have the children turn to Worksheets 1 and 2. Have the children complete the worksheets. When all are finished, compare answers.





Activity C

Tell or read the story, "The Problem of the Two Inchworms," to the children. Note that the children should have an opportunity to give their ideas on the problem before the spider gives his solution.



12

31

THE PROBLEM OF THE TWO INCHWORMS

One, two, three! A thin, green inchworm arched his slender body in a tiny loop, as he made his way along a brown twig.

And one, two, three -- again! Another little inchworm arched his body and traveled along the other side of the same twig. Together they inched all the way to the end of the twig. Then, resting there, the first inchworm said, "Well, friend, this twig is fourteen loops long."

"Wrong!" said the second inchworm. "I was counting, too, and I happen to know that this twig is sixteen loops long! I arched my back and straightened my body out sixteen whole times so the twig just has to be sixteen loops long."

The first inchworm measured the twig with his body again, "Fourteen!" he insisted. "In fact it is a little less than fourteen." Then the two inchworms began to argue. They argued so much that a spider on a near-by twig stopped to listen. He heard the second inchworm say, "This is most peculiar. You and I both started at the bottom of the twig and we both went the same length, and yet when we reached the end, you had only made fourteen loops, but I had made sixteen. What a problem!"

"I think I know the answer," the spider said.

"You do?" the inchworms asked. "Well, tell us, please! This is too much of a mystery!"

(Stop here to allow the class to suggest solutions of the mystery.)

"It's very simple, really," the spider said, "one of you must be longer than the other. It must be you," he continued, pointing a leg at the first inchworm. "You didn't need to arch your back and straighten out as many times, because you just happen to be a slightly longer inchworm than your friend."

The two inchworms found this hard to believe. They stretched out side by side to see, and sure enough, one of them was longer than the other. Then the spider hurried away on some errands of his own, and the inchworms looked around for another twig to measure.

Activity D

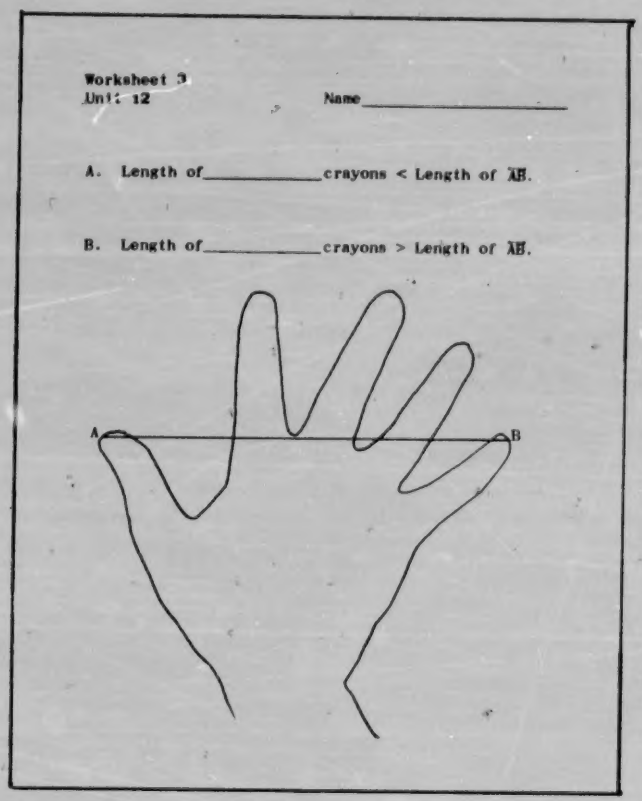
Discuss with the class how a single reference unit can be used in measuring a length. Draw a line segment on the chalkboard.

LET'S MEASURE A LINE SEGMENT ON THE CHALKBOARD IN ERASER LENGTHS.

HOW CAN WE USE JUST ONE ERASER? (We can pick it up and put it down again.)

LET'S COUNT HOW MANY TIMES WE HAVE TO USE THE ERASER.

Emphasize that the reference object must be laid down along the line segment. Make marks at the end of the reference object each time it is placed down. Also caution the children that the end of the reference object must be placed directly on top of the mark just made.


Worksheet 3
Unit 12

Name_____

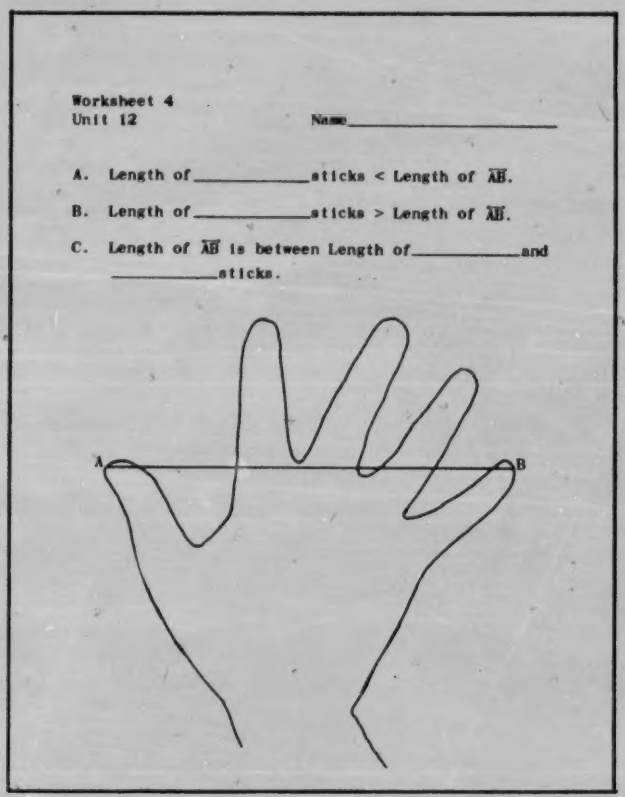
A. Length of_____crayons < Length of $\overline{AB}$.

B. Length of_____crayons > Length of $\overline{AB}$.

Have the children turn to Worksheet 3. They should remember from their studies in Unit 10 that the symbol for a line segment is expressed by two labeled points, such as A and B, with a bar above them. Thus "line segment AB" is written as $\overline{AB}$. Read the instructions to the children and then have them use crayons to measure the pictured handspan (line segment AB) and record their results. There should be variations in the lengths of crayons selected. If you think it necessary, a transparency could be made of the worksheet and it could be completed by the class before the children do the work independently. Record various results on the chalkboard. When the children raise questions about the variation in results, ask:

WHY DIDN'T ALL OF YOU GET THE SAME RESULT?

Their discussion and suggestions should lead them to see that the chief cause of the difference was the use of crayons of various lengths. This idea can be reinforced by having them exchange crayons and repeat the measurement. Again the need for using a standard length as reference unit should be discovered.



For the second part of this activity, select a standard unit which will be used by the whole class to measure the length of the line segment on Worksheet 4. Since all the children will use reference objects which have essentially the same length, choose an object of such a kind that each child has one. This could be a Popsicle or paste stick, a Minnebar of a certain color, or an unused crayon.

The results of these measurements should be compared and should show much closer agreement. Possible reasons for variation include mistakes in counting and lack of skill in measuring. If the length of the line is very close to some whole number of standard units, the children may still disagree. Have each child repeat the measurement several times (perhaps starting at the other end) to find out whether he gets the same results each time. This should improve technique and lead the children to the good practice of repeating any measurement.

Supplementary Activity A on page 59 is appropriate at this time.

Lesson 3: MEASURING WITH PAPER CLIP CHAINS

This lesson gives further practice in measuring with a standard unit and recording the results. Paper clips are used as standard reference units. Some formal instruction will be necessary to teach the technique of measuring and the methods of recording the measurements. For each length that is measured, two paper clip chains are needed which differ in length by one paper clip: one of the chains will be shorter than the object being measured and the other chain will be longer or appear to be the same length as the object.

MATERIALS

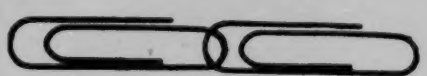
-- for each child --

- 20 paper clips all of one size (A convenient size is #1, which is about $1\frac{1}{4}$" long.)
- crayon
- Worksheets 5 and 6
- strip of heavy white paper, 11" x $1\frac{1}{2}$"

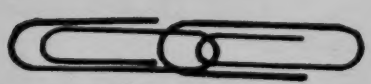
PROCEDURE

Activity A

Give each child about 20 paper clips. Leaving some single units, show the children how to assemble these into chains of 2, 3, 4 and 5 units. Putting the paper clips together into chains presents problems for some children who lack dexterity. Care should be taken that the big outside loop of one paper clip is joined to the outer loop of the next and that the inside loops are not coupled. (See the diagrams.)



Correct



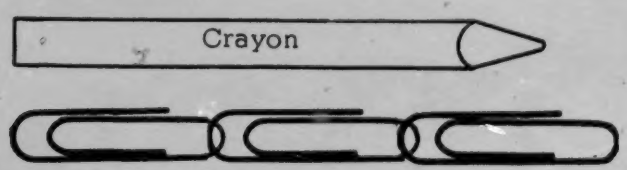
Incorrect

Have each child select a crayon.

FIND A PAPER CLIP CHAIN THAT IS JUST SHORTER THAN YOUR CRAYON.

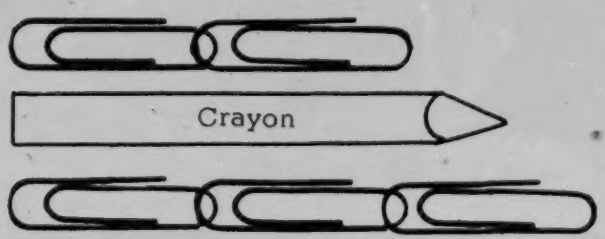
FIND ANOTHER PAPER CLIP CHAIN THAT IS JUST LONGER THAN YOUR CRAYON.

Two paper clip chains are used for the measurement to illustrate clearly that the ends of the paper clips are to be used rather than some intermediate point. An illustration will help explain this. Suppose a crayon is being measured with one paper clip chain:

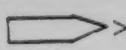
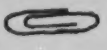


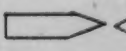
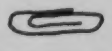
The children are likely to say the crayon is three paper clips long because its right end falls on the third paper clip. It should not be called three paper clips long unless the right end of the crayon appears to be almost even with the end of the third paper clip. Ask the children which is longer, the crayon or the chain of three clips.

By using two paper clip chains as shown, the children should acquire a correct technique for measuring and describing the result.



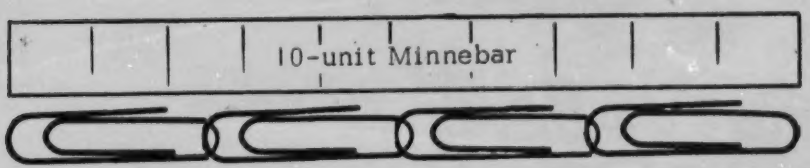
Discuss how the results of the comparisons can be recorded, putting an example on the chalkboard and explaining how to read it. For example, the comparison above should be written,

Length of  > Length of 2 

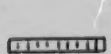
Length of  < Length of 3 

and read, "The length of the crayon is greater than the length of two paper clips and the length of the crayon is less than the length of three paper clips."

Also use a demonstration situation in which the length of a crayon (or another object) appears to be the same as that of a chain of paper clips.



This should be written,

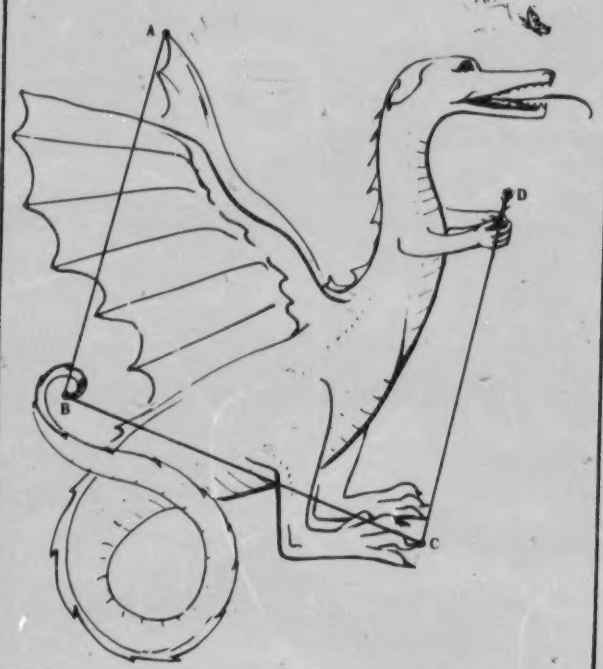
Length of  ≐ Length of 4

and should be read, "The length of the object appears to be the same as the length of four paper clips."

Give the children repeated practice with the symbolism before they use it in their individual records.

Worksheet 5
Unit 12

Name _____



Worksheet 5, continued
Unit 12

Name _____

On Worksheet 5 use a ruler to draw $\overline{AB}$, $\overline{CD}$, $\overline{BC}$. Put the numerals in the blanks.

1. Length of $\overline{AB}$ > Length of 4 paper clips, and
   Length of $\overline{AB}$ < Length of 5 paper clips.
   Length of $\overline{AB}$ is between Length of 4 paper clips
   and Length of 5 paper clips.

2. Length of $\overline{CD}$ = Length of 4 paper clips.

3. Length of $\overline{BC}$ > Length of 4 paper clips, and
   Length of $\overline{BC}$ < Length of 5 paper clips.
   Length of $\overline{BC}$ is between Length of 4 paper clips
   and Length of 5 paper clips.

Activity B

Have the children turn to Worksheet 5. Read it to them. They are to use paper clip chains to measure and record the lengths of the line segments they draw on the picture of the dragon.

Each child's measurements should be checked and the activity discussed with the class.

To make the longer chains they need, the children may add extra clips to the ones already assembled or hook some chains together. This is an opportunity to reinforce addition. (Example: 5 + 3 = 8.)

Worksheet 6
Unit 12 Name_____

Trace around your hand in the empty space. Draw a line segment from the tip of your thumb to the tip of your little finger. Label it $\overline{AB}$. Measure $\overline{AB}$ with your paper clip chain. Record your results on the lines.

_____

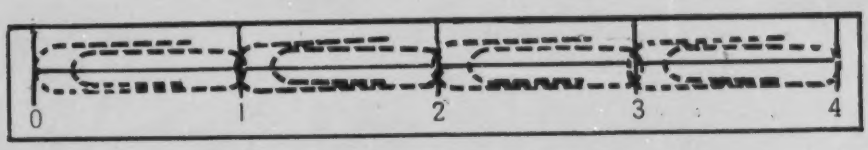
_____

Activity C

Have the children combine all their small paper clip chains into one large one. Then have them use this long chain to complete Worksheet 6. Each child is encouraged to record his own results on the bottom of the worksheet. Many children may find this difficult to do. Accept any reasonable recording the children do.

Activity D

Finally, by making paper clip rulers, some children may solve the problems of trying to keep the paper clip chains straight and of counting the paper clips. Give a strip of heavy white paper to each child. They can draw a line segment on the paper and number marks along the line according to the spacing of the clips.



It might be interesting for the children to combine several paper clip chains into a longer one to measure some larger objects, such as a table or the length of the classroom.

Some discussion might take place about whether paper clip chains could be used to measure other lengths such as the length of the school building, the length of the flag pole, or the distance from home to school.

Supplementary Activity B, on page 59, is appropriate at this point.

Lesson 4: DRAWING LINE SEGMENTS

This lesson develops the child's ability to:

- Reproduce a length from a given number of reference objects.
- Understand that a common reference unit is needed to communicate meaningfully about lengths.

In Lesson 3 the children described lengths in two ways: (1) as being between two paper clip units in length or (2) as being approximately equal to some whole number of units. In this lesson the children should draw line segments that can be described in both ways.

Drawing a line segment of a specified length is more difficult than measuring one that is already drawn. Whether the children draw the lines freehand or use a ruler, you need not be concerned about how straight the lines are. The children drew line segments in Unit 10, but this lesson is different because it requires them to select their own endpoints and to use a reference object to determine the lengths.

MATERIALS

-- for each child --

- chain of 6 (or more) paper clips
- Worksheet 7
- drawing paper (optional)

-- for each pair (optional) --

- 3 reference objects of different lengths (e.g., ruler, eraser, paper clip or crayon)
- sheet of plain paper large enough for drawing line segments the length of any of the selected objects

PROCEDURE

Activity A

Have the children turn to Worksheet 7. Discuss it with them before they attempt to draw the line segments of given lengths.

Check their results as the children work. You need not worry about slight variations in length if the work is fairly accurate.

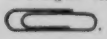
Worksheet 7
Unit 12 Name_____

Draw these line segments.

A length > 3 .

A length < 4 .

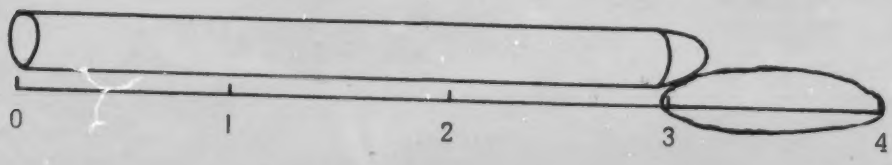
A length that is between the length of 3 and 4

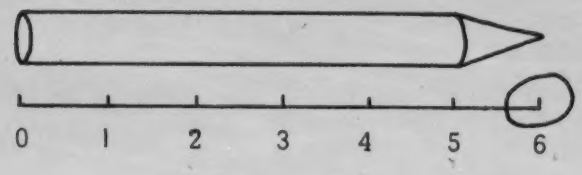
 .

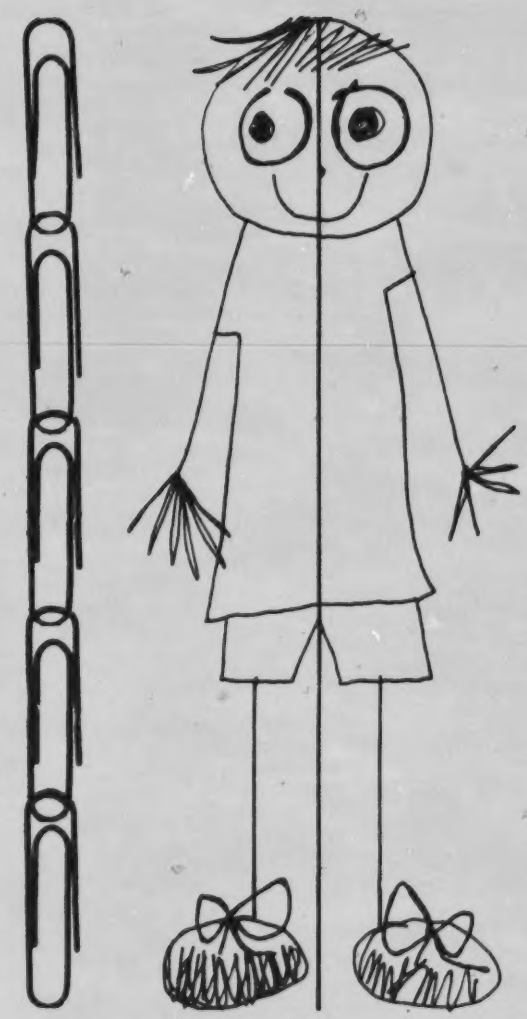
A length = 5 .

Activity B (Optional Game)

Arrange the children in pairs facing each other across a desk or table. Give each pair a choice of three reference objects (e.g., ruler, eraser, paper clip or crayon) and a large sheet of plain paper. One child should close his eyes while the second child chooses a reference object and uses it to measure some other item taken from his desk. (This item should be small enough to fit on the paper.) Then the second child puts the object he has measured away in his desk, or out of sight, and says "Ready!" He tells his partner (the first child) the length of the object in his chosen reference unit. For example, he might say, "The object I measured is between three and four erasers long." The first child then draws a line segment four erasers long. Then he draws a closed curve around the interval, or around the numbered mark (if the length appears to be the same as that of a numbered mark). See diagram below and that on the top of the next page.







When he is done, he asks, "Is your object this long? What is it?" Then the children put the object on the line segment to check. For the next round, they exchange roles.

Modifications can be devised for this game, such as establishing a scoring method before the children exchange roles. Or one child may ask another which of his reference objects he used in order to have an object come out to be so many units long.

Activity C (Optional Art Activity)

Give a sheet of drawing paper to each child. Tell the children to draw a line that appears to be five paper clips long (or any other length you might choose). Now have them use the line segment as the basis for a drawing.

Lesson 5: COMMON UNITS OF LENGTH

This lesson acquaints the children with common units of length (inch, centimeter) and provides practice in measuring with rulers and in recording the measurements.

The common units which we use for measuring have interesting histories. Mankind's need for standard units of length has evolved over centuries in a way quite similar to the development which has been proceeding in your classroom. The early inch was the width of a man's thumb, or that of three barley corns; the foot was the length of a man's foot; and the yard was the distance from the nose to the finger tips of an outstretched arm. Since these units of length varied from one person to another, it became necessary to establish the common standard units of length which we use today. A history of measurement can be found in the booklet, "A World of Measurement" by Johnson and Glenn from the series, Exploring Mathematics on Your Own, Webster Publishing Company, 1961. A story to read to children is How Big is a Foot? by Rolf Myller, Atheneum Press, New York, 1962.

The centimeter is a standard unit of length which is about 0.4 inches long. It is one of the length units in the widely used Metric System. A meter is 100 centimeters long. One tenth of a centimeter is called a millimeter. Four inches are about 10 centimeters, and a foot is about 30.5 centimeters.

While the centimeter and other units of the Metric System are not in common use in this country, they are very likely to be in the not-too-distant future. The chief reason for introducing their use is to give the children experience with various units of measure, thereby freeing them from commitment to any one set. It is not intended that the children should become involved with converting measurements from one set of units to another.

After using paper clip chains for measuring, the children will find that, for objects less than twelve inches in length, the rulers are easier to use.

A ruler can be thought of as a representation of part of a number line, or as a whole set of reference objects one inch, two inches, three inches, etc., in length. A measured length may

be between two of these — ending between two marks on the ruler. (This is the kind of comparison the children had in Lesson 1.)

The children practice measuring lengths of objects, then drawing lines of certain lengths.

Measuring objects that are longer than the rulers provides excellent motivation for practice in using the number line for addition. For example, to measure an object about 21 inches long with a 12-inch ruler will require that the children be able to add 12 and 9 or that they devise some way of counting to obtain 21. A yardstick (as a longer piece of the same number line that is used on the foot ruler) can be employed to show that 12 + 9 = 21, as well as for other addition combinations. This is developed in detail in Unit 13.

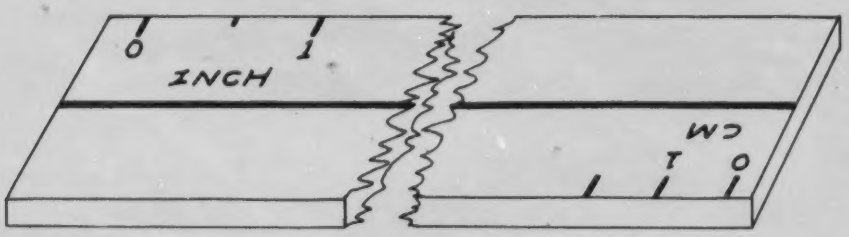
MATERIALS

-- for each child --

- ruler, with inches, half-inches and centimeters indicated
- Worksheets 8 through 10
- set of Minnebars

PREPARATION

You may wish to print "inch" and "cm" (centimeter) on each ruler as indicated below to familiarize the children with the units involved. Or they may be able to label the scales themselves.



PROCEDURE

Activity A

Read the story, "The Measuring String," to the children. Encourage discussion of the inconvenience of not having common standard units of measurement. Perhaps the children would enjoy drawing pictures, in their free time, of houses the way they would look if the carpenters had no way of measuring, or of people with hats, clothing and shoes that do not fit.

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Story by Elaine Vogt

Illustrations by Sonia Forseth

THE MEASURING STRING

Henry was a boy who lived a long time ago in a town of friendly, foolish folk. They had never learned how to measure anything, so their clothing and their homes were very strange. Even the streets zigged and zagged in all directions.

This town had every sort of shop that such a town needed. It had a hatmaker, a tailor, a shoemaker, and a grocer.

The shoemaker made plenty of shoes, but there was something peculiar about them. He always made them to fit his own feet! The shoes would fall off the babies' feet because they were too big. But men with big feet had to squeeze and squirm and wriggle into their shoes. Their feet hurt so much that the men could hardly wait for warm weather so they could go barefoot and leave their tight shoes at home.

The hatmaker made all the hats, caps and crowns, but there was something peculiar about them too, because he made them to fit his own head — which was rather large. The king and queen looked ridiculous; their crowns came down to their ears. And little boys and girls had to cut holes in their hats so they could see where they were going.

HATMAKER

Going shopping could be very confusing. All the hats were the same size and all the shoes were the same size, but all the loaves of bread were a different size! The first time Henry went to the bake shop for his mother, he had a terrible experience. When he said, "My mother wants an armlong of bread," the baker measured the length of the bread by Henry's arm, which was pretty short. Henry's mother was very upset. "That's not an armlong of bread," she said and she scolded him quite thoroughly. After that she always sent Henry's father or a favorite cousin who had very long arms.

Henry's feelings were hurt, and he thought -- perhaps more than he should -- about the difference between his arms and his cousin's. Every year he thought his arms were growing longer, but his mother never did send him for any more bread.

BAKERY

Finally Henry grew to be as tall as any man in the town. And then a little taller. His feet grew much larger too, and so did the size of his head. Soon after he grew up, people began noticing that Henry seemed much smarter than anyone else, so they elected him to be Hizzoner, the Mayor of the Town. There were many advantages to being the Mayor, but even so, Henry had many things that troubled him. The worst thing was the fact that his feet always hurt terribly because they were squeezed into tight, tight shoes. And it was very annoying, too, to have his hat fall off his head constantly, and to have clothes that were too short and too tight.

One day, he was admiring his new fur collar in the mirror and wishing that it was not quite so tight around his neck. He turned this way and that and held his arms out wide. My, his arms had grown long! Of course his mother wouldn't think of sending the Mayor out to buy bread, but he just wondered how big a loaf he would get if he asked for an armlong of bread now.

He remembered how bad he used to feel when his arms were short, and he thought how nice it would be if all the little boys could go to the baker for their mother and get the same big, long armlong of bread. Then he saw a piece of string hanging from the hem of his coat. It gave him an idea. He grabbed an end of the string and measured the distance from the tip of his nose to the end of his outstretched finger with it.

He sent the Town Crier out to tell all the townspeople the good news.

"Hear ye! Hear ye! From now on everyone will get the same size armlong of bread. Hizzoner Henry has the longest arm in our town, and he has given the baker a piece of string as long as his arm. From now on the baker will bake all loaves of bread as long as Hizzoner's arm!"

Wherever the Town Crier went, the people shouted "Hooray!" The littlest boys were especially happy because they would no longer be scolded for bringing home the littlest loaves of bread.

Well, Henry was very proud of himself! He sat back in his chair and looked out the window at the big and little people trotting by, all carrying the same size armlongs of bread. He had cut another string to the same measure and he loved to sit and play with it while he thought about other town problems that he might try to solve.

He leaned back in his chair playing with his string, and put his aching feet up on his desk. (They ached a little less that way.) Well, now, there was a problem to solve! Henry dangled his string against his feet. He noticed that his armlong of string was just exactly as much as three lengths of his foot.

Hooray! Now that problem was solved! He sent a messenger flying to the shoemaker to make a pair of shoes just one third of an armlong. Shoes that would fit Hizzoner!

Oh, they felt good! And, as a good Mayor, he wanted to share this pleasure with his townspeople. So he sent the Town Crier out again, and the people went running to the shoe maker for new shoes that would fit just right.

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But there was something wrong! The shoes still fell off the children's feet -- and now they fell off some of the grownups' feet, too. And for just a few people, the new shoes were still too tight.

The people came to Hizzoner the Mayor. "Help us," they said. "You are supposed to solve all our problems, but you didn't solve this one."

Well, Henry sat and sat, and thought and thought. "An armlong of bread is an armlong of bread," he said, as he stretched his string out full length -- "and a footlong makes a perfect shoe," he said, as he folded the string into three parts. "But not for everyone," he added sadly. "How can we solve this problem?" He twisted the string round and round his thumb while he thought. Around and around, once, twice, three times. Four times around his thumb was as many times as the footlong would go. And twelve times around for an armlong.

He thought and thought for a long time, and kept twisting and counting. Suddenly he realized that he got the same number of twists every time. "Hm," he thought, "I wonder if I could use that piece of information somehow."

He tried to twist the string around his head, but that didn't come out even. His head was less than an armlong around, but more than a footlong. Maybe he could count the number of thumb-twists of string that could reach around his head? He did it five times, and each time it came out the same. (Well, almost.)

"Hm," he thought again. "Hm." He remembered the problem he used to have with armlongs of bread when he was a little boy. Not everyone's arm was the same length. Suppose he were to ask the hatmaker to make him a hat that was eight thumb-twists around. But suppose the hatmaker had a skinny thumb? (Which he did!)

At last he had it! Hizzoner Henry jumped up and danced around the room, waving his string in the air. "We always have to use the same string," he shouted. He leaned out the window and shouted to the townspeople, "We always have to use the same string!"

They looked at each other. They were very puzzled. But Henry wasn't Hizzoner the Mayor for nothing. Henry was the only wise man in town. So they nodded, "Yes, we always have to use the same string."

Henry sat down and marked an armlong of string off into three footlongs. And then he marked each footlong off into four thumb-twists. He marked a string for the shoemaker and a string for the hatmaker and a string for the tailor. And from then on, whenever anyone wanted shoes to fit, if he had very big feet he asked for shoes one footlong and one thumb-twist long. Or if he were very little, he asked for shoes just two thumb-twists long.

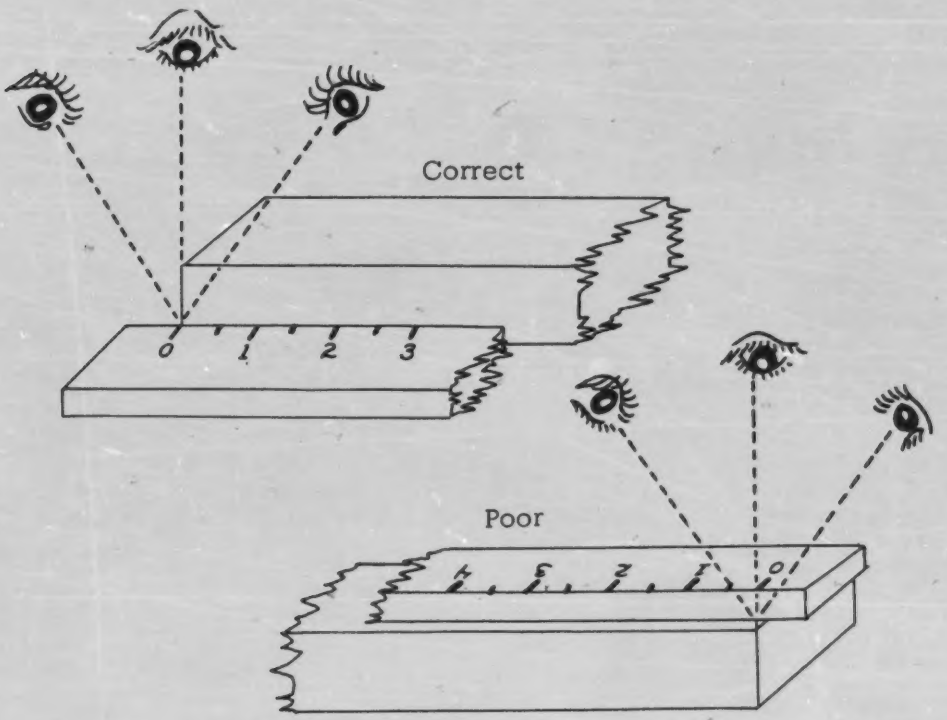
And believe it or not, there were no more aching feet in the whole town! And everyone -- even the king and queen -- looked neat and trim in hats, coats and shoes that fit them just right. And they had a portrait painted of Hizzoner Henry in his new, well-fitting clothes, holding a gold armlong of string with the footlongs marked with diamonds and a ruby for each thumb-twist!

Activity B

Give each child one of the foot rulers provided in the MINNEMAST kit.

When introducing the ruler for length measurement, take time to familiarize the children with certain techniques that will improve their ability. The MINNEMAST rulers have purposely been designed with the zero not at the end. This gives children the idea that markings are more accurate to use than the end of the ruler, because corners soon become worn, thus introducing error.

It is a good practice to place the ruler in such a position that the scale is as close as possible to the object being measured otherwise the thickness of the ruler causes parallax errors (variations in reading due to the angle from which the reading is taken).



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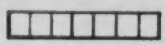
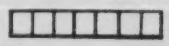
The intermediate marks on ordinary rulers are undesirable at this level since they require an understanding of fractions to be meaningful. Too many subdivisions of the scale also tend to be confusing. Some time after the rulers are distributed, show the children how to measure in inches and in centimeters and how to record these measurements (some with two < symbols, some with ≐). Some of the Minnebars have lengths which come out "between" on one scale and "about the same as" on the other.

Have the children measure and record the length of the turquoise blue Minnebar in inches.

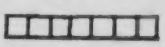
HOW DID YOU RECORD THIS MEASUREMENT?

Length of 3 inches < length of 

Length of 4 inches > length of 

Verbalize this recording with the children. Now ask them to measure the same Minnebar in centimeters.

HOW CAN WE RECORD THIS MEASUREMENT?

Length of 9 cm ≐ length of 

Again read the recording with the children.

Activity C

You or the children select objects whose lengths are to be measured with the foot ruler. The classroom contains many appropriate objects:

Length of the top and the side of the worksheet.
Length, width, thickness of books, boxes, blocks, etc.
Child's handspan, palm width, first thumb joint.
A chain of 5 paper clips.

Lengths of edges of geometric figures: triangle, square, rectangle, etc.
Diameters of circles: cups, dishes, wheels. (These may be difficult, but worth trying.)
Dimensions of desk top, table top, shelves, doors, windows.
Heights of children.
Lengths of Minnebars.

The number of objects your class should measure with each standard unit, as well as the amount of time the children should practice measuring and recording, are left to your judgment. You may decide to use Supplementary Activity C (which follows Lesson 7) now or intermittently through the rest of the school year.

Worksheets 8 and 9 give the children practice in the measuring and recording of line segments in both inches and centimeters. Read Worksheets 8 and 9 with the children. Then have them complete the work with as little help from you as possible.

Worksheet 8
Unit 12

Name _____

Worksheet 8, continued
Unit 12

Name _____

On Worksheet 8, use a ruler to draw $\overline{AD}$, $\overline{CB}$, and $\overline{CD}$. Fill in either A or B.

1. Length of $\overline{AB}$ in inches:

   A. Length of $\overline{AB}$ < 8 inches.
   Length of $\overline{AB}$ > 7 inches.

   B. Length of $\overline{AB}$ = _____ inches.

2. Length of $\overline{CB}$ in inches:

   A. Length of $\overline{CB}$ < 5 inches.
   Length of $\overline{CB}$ > 4 inches.

   B. Length of $\overline{CB}$ = _____ inches.

3. Length $\overline{CD}$ in inches:

   A. Length of $\overline{CD}$ < _____ inches.
   Length of $\overline{CD}$ > _____ inches.

   B. Length of $\overline{CD}$ = 9 inches.

Worksheet 9
Unit 12 Name_____

Measure the line segments you drew on Worksheet 8 again. This time measure with centimeters. Fill in either A or B.

1. Length of $\overline{AB}$ in cm:

   A. Length of $\overline{AB}$ <_____cm.
   Length of $\overline{AB}$ >_____cm.

   B. Length of $\overline{AB}$ = 19 cm.

2. Length of $\overline{CB}$ in cm:

   A. Length of $\overline{CB}$ <_____cm.
   Length of $\overline{CB}$ >_____cm.

   B. Length of $\overline{CB}$ = 12 cm.

3. Length of $\overline{CD}$ in cm:

   A. Length of $\overline{CD}$ <_____cm.
   Length of $\overline{CD}$ >_____cm.

   B. Length of $\overline{CD}$ = 23 cm.

Worksheet 10
Unit 12 Name _____

Draw line segments of these lengths:

_____

Length > 1 inch

_____

Length < 2 inches

_____

Length between 1 inch and 2 inches

_____

Length = 4 cm.

Activity D

Worksheet 10 asks the children to draw line segments of several lengths. This should present less difficulty than the similar work of Lesson 4, as the ruler is used both as a straightedge for drawing the lines from the zero mark and for measuring the lengths drawn.

Lesson 6: VARIATION IN LINEAR MEASUREMENT

The children will use their shoes in this lesson to get more practice in measuring lengths with standard units and to learn to make a histogram of the results.

A histogram is one way of summarizing the results of many similar measurements. Columns are built up over a number line made up of intervals that correspond with those of the measured shoe lengths. The height of each column corresponds with the number of shoes found to have a length belonging in that interval.

The children had some preparation for the study of histograms in Lessons 6 and 7 of Unit 8, where objects were sorted into subsets and then lined up in columns. You may not want to make a histogram of the actual shoes, as it is easier to use a grid.

MATERIALS

-- for the class --

- 12" x 12" pegboard
- golf tee for each child
- other materials for making a histogram (optional, see preparation)

-- for each child --

- inch and centimeter ruler from Lesson 5

PREPARATION

Insert a golf tee from the back in a hole at each corner of the pegboard so it can be propped up on one edge in the chalk tray. You may have to line up two or three pegboards in this manner if you plan on having the children measure their shoes in centimeters. If they will use inches, one pegboard is enough. Put masking tape horizontally across the boards near the bottom. Draw in a number line, making integers between the vertical columns of holes. This line should start with zero, even though there will be many unused columns.

You might prefer to make a transparency of a histogram to use on an overhead projector. This could be saved for the work with volume in Lesson 13.

You could also draw a grid on the chalkboard and draw chalk counters, or you could use the flannel board and clinging counters. The important thing in making a grid is to have the lines evenly spaced.

PROCEDURE

Ask each child to measure the length of his shoe with a centimeter ruler; or, if the children need review for numbers less than twelve, have them use the inch rulers. They may measure the length of their shoes in any of the following ways:

1. Trace around the shoe on a piece of paper and measure the resulting diagram.

2. Step on the ruler, being sure the heel of the shoe is at zero.

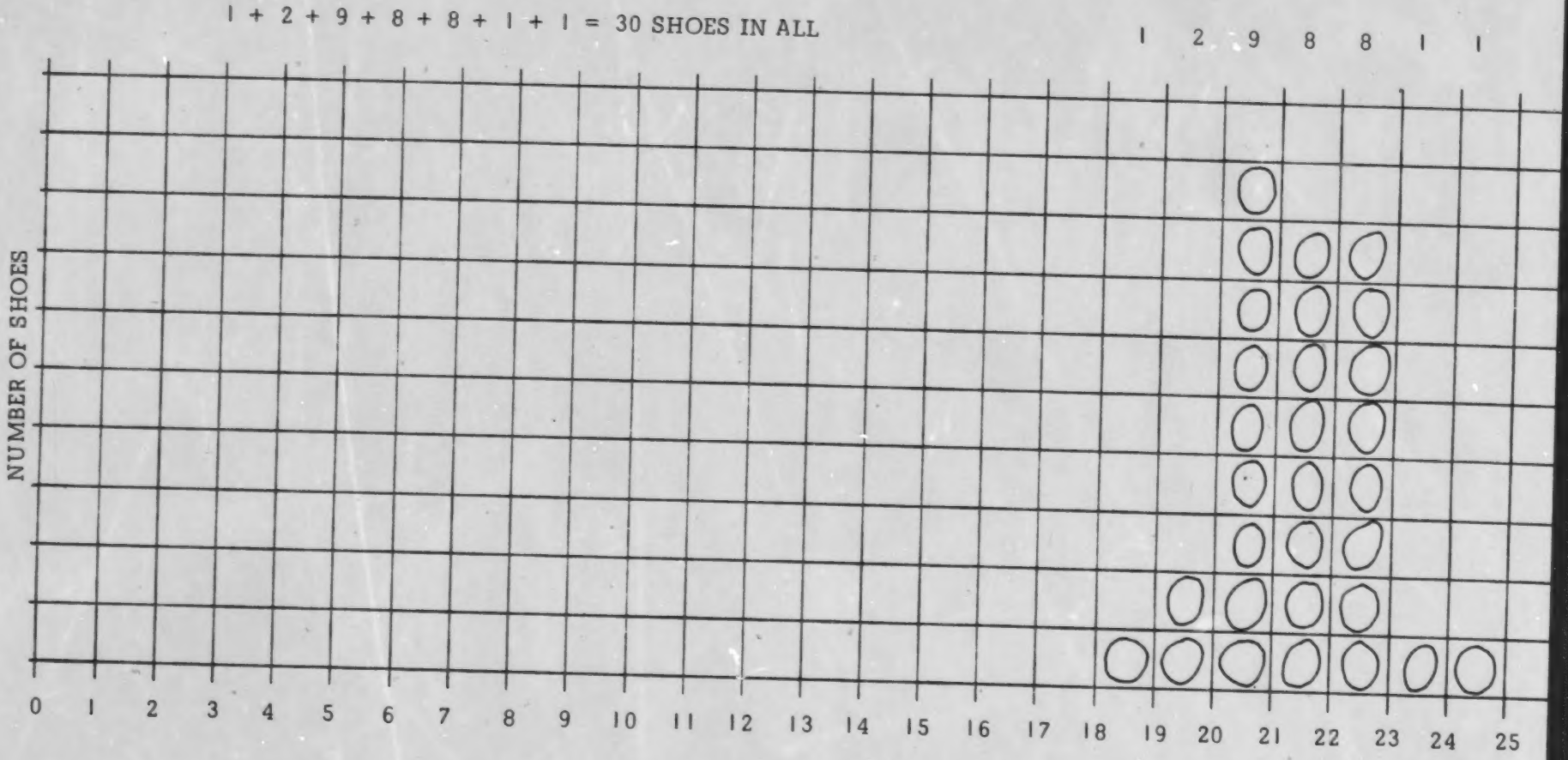
3. Remove the shoe and take the measurement along the bottom. This may be a practical method to use for outer boots, if all the children have them.

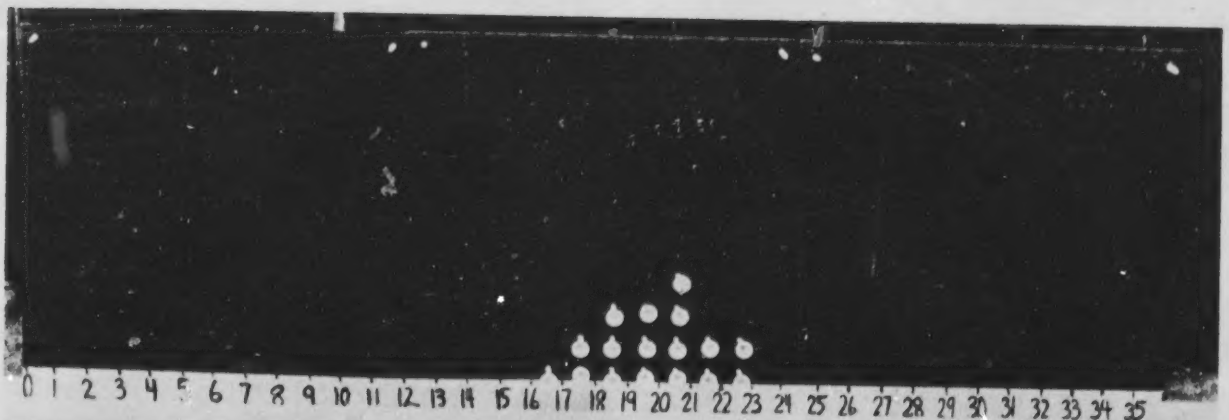
Ask each child to give you the result of his measurement in the interval form, e.g., "between 19 and 20 cm." If a child insists that his result is precisely a whole number, include it in the next higher interval. Record the results in a list on the chalkboard. Then ask the children if they can think of a way to make the record easier to understand or to see all at once.

Bring out the pegboard you have prepared for the histogram. Have each child again give you the results of his measurement.

As a child again reports his shoe length, say between 20 and 21 cm (or precisely 20 cm), he puts a golf tee between 20 and 21, in the lowest empty hole above the number line.

TYPICAL HISTOGRAM OF SHOE LENGTHS





If you think the children would profit from it, this histogram could also be put on a chart on the chalkboard or on a large sheet of paper. Draw or mount an appropriate grid on the chalkboard and have each child draw an 0 or place a counter in the proper column to indicate the interval in which the length of his shoe falls.

Lead the children to discuss the significance of the record — that all the shoe lengths fall between certain limits and that most of the shoes are about a certain length, between 20 and 21 centimeters in our illustrative data.

At the top of each column, have the children record the number of shoes of that length — the number of counters in the column. Ask how many measurements were recorded in all. They may get this result by counting all the counters. Some may be ready to discover that they can add the number of tees in all the columns. They should be encouraged to do this in any way they wish: by using Minnebars, a number line slide rule, making their own number line and counting jumps or by any other method.

If you feel that your class will gain from more activities like this, repeat the above but substitute handspan, length of forearm (tip of finger to elbow), etc., for shoe length. Any rulers or other length scales that give a variation in measurement can be used.

Save the histogram apparatus for use in Lesson 13.

Lesson 7: PERIMETERS

In this lesson the children measure curves consisting of more than one line segment and boundaries of special shapes to obtain the perimeters.

In anticipation of the work on area in Section 2, this lesson begins the distinction between perimeter (the length of the boundary of a region) and area (the size of the region enclosed by the boundary). The derivation of the word emphasizes its significance:

> peri, coming from the Greek language, means "around"
> meter, also from the Greek, means "to measure"
> perimeter is found by measuring around a closed curve

The activities start with the measurement of line segments, then combine such measurements for a broken line, and finally use them for a combination of segments which form a closed curve — the boundary of a region.

MATERIALS

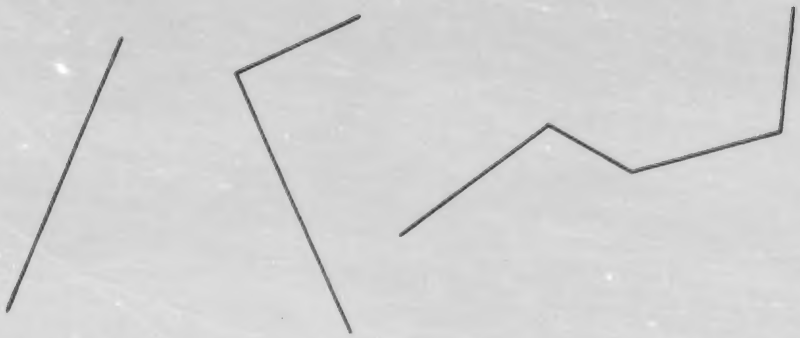
-- for each child --

- Worksheets 11 through 13, ruler
- property blocks
- cards illustrating simple open curves (optional)
- string or narrow paper strips, or both

PROCEDURE

Activity A

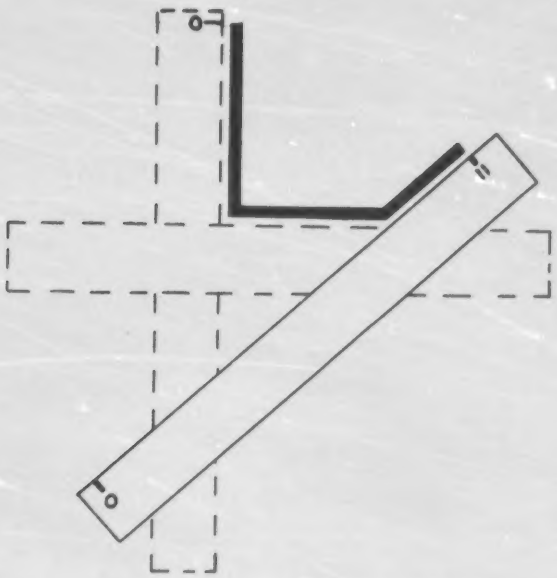
On the chalkboard draw a straight line segment which is less than ten rulers long. Measure the segment using the length of a ruler as your unit. Draw other lines in many different directions. Draw several zigzag lines which change direction. Give the children a chance to measure these segments.



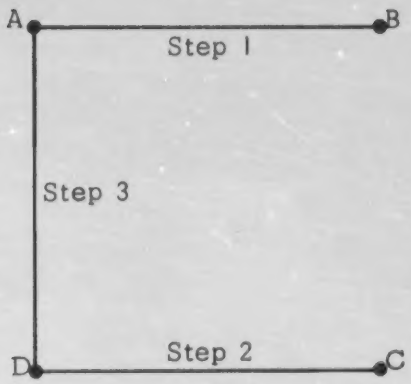
WHAT ARE THESE LINES CALLED? (Open curves.)

How can the children obtain the length of a zigzag curve? Perhaps by finding the sum of lengths of the segments — which may be a reasonable request, but not if each length comes out between whole numbers of units. If a child suggests this method, have him try it to discover the difficulty for himself.

Instead the children may use the ruler to find the sum: one segment is measured, then the ruler is pivoted. The only reading of the measurement that needs to be read is the final result.



Draw four points on the chalkboard and label them: A, B, C, D. Using a ruler, connect points A and B. Measure and record the length of line segment AB. Repeat the procedure for points C and D. Discuss with the children the type of curves we have considered so far (two simple open curves.) Now, connect points D and A.



Open curve

Closed curve

IS THERE ANY CHANGE IN THE TYPE OF CURVE? (Now it is one simple open curve.)

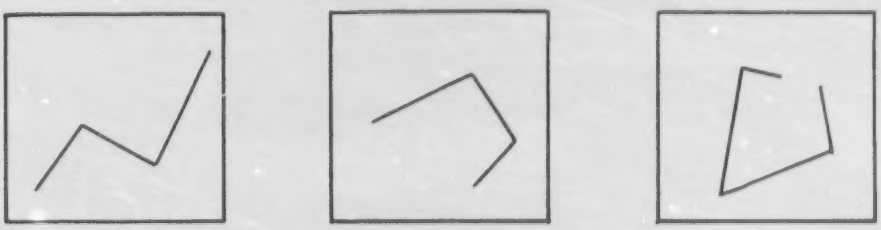
Connect points B and C.

WHAT TYPE OF CURVE DO WE HAVE ON THE CHALKBOARD NOW? (Simple closed curve.)

HOW MANY UNITS LONG IS THIS CLOSED CURVE?

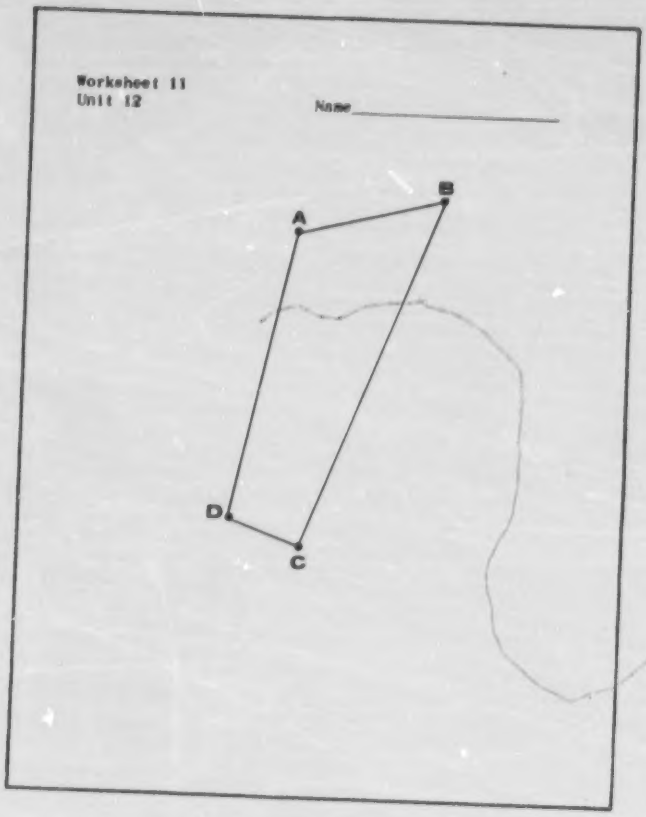
At this point use the word "boundary" when you talk about a given closed curve. Explain this in a discussion with the children about the measurement of the length of a closed curve or boundary of a region. Tell them there is a special name for this length — perimeter. Reinforce this idea that the length of the boundary of a region is the perimeter. See if the children can develop in their own words a meaning for perimeter (length all the way around a boundary).

You may want to prepare cards such as those shown below. These may now be passed around. The children can measure the length of the simple open curves, and then exchange cards and compare results.



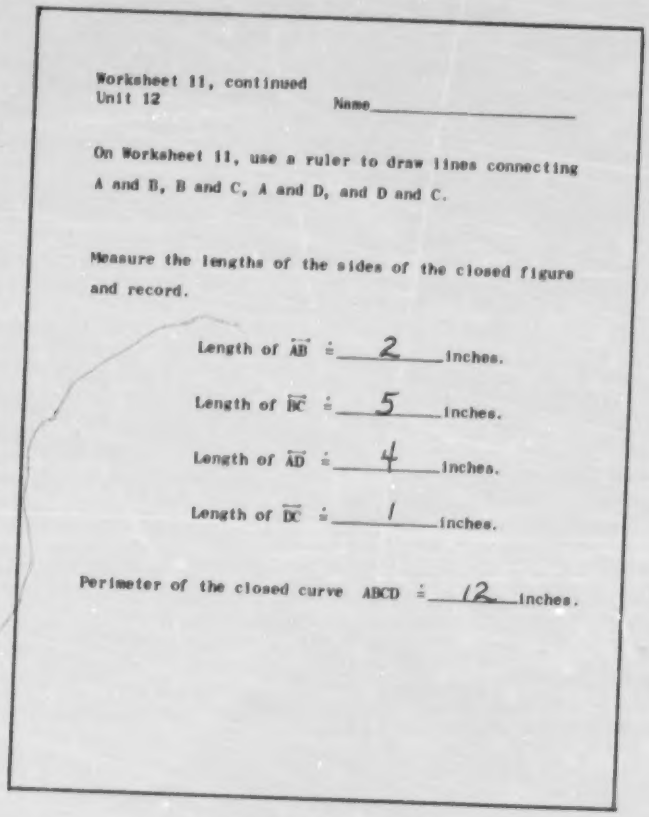
Activity B

Have the children turn to Worksheet 11. Read the instructions with them and have them complete the worksheet.



Worksheet 11
Unit 12

Name_____



Worksheet 11, continued
Unit 12

Name_____

On Worksheet 11, use a ruler to draw lines connecting A and B, B and C, A and D, and D and C.

Measure the lengths of the sides of the closed figure and record.

Length of $\overline{AB}$ ≐ 2 inches.

Length of $\overline{BC}$ ≐ 5 inches.

Length of $\overline{AD}$ ≐ 4 inches.

Length of $\overline{DC}$ ≐ 1 inches.

Perimeter of the closed curve ABCD ≐ 12 inches.

Activity C

Use Worksheet 12. Ask the class to use rulers to measure, in inches, the perimeters of the four shapes found on the worksheet. Have the children record their results on the line near the shape. They can figure their answers by pivoting rulers, by counting, by tallying or by addition of two or more numbers.

After the children have completed the measuring on the worksheet, use questions similar to the following to discuss the information they have found.

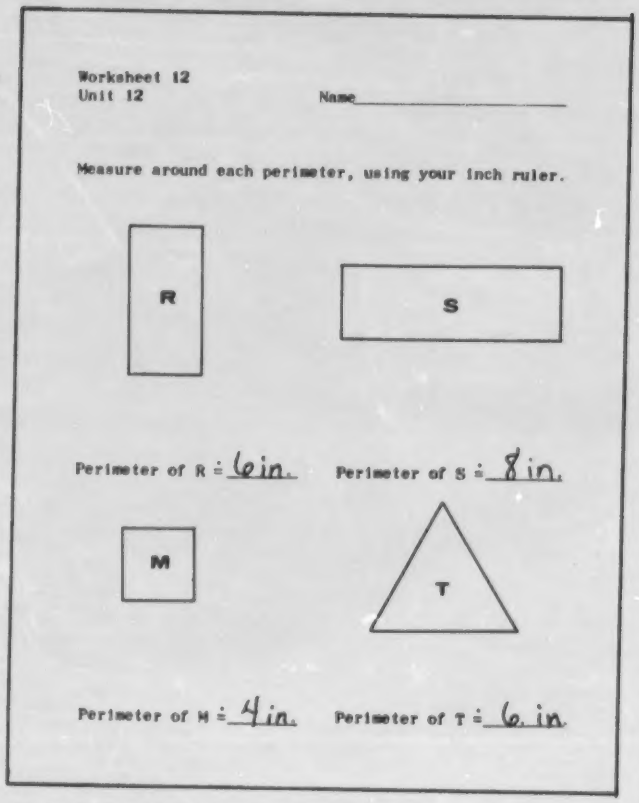
WHAT DO WE CALL THESE CURVES? (Closed curves, rectangles, squares, triangles.)

WHICH SHAPE HAS THE LONGEST BOUNDARY, THE GREATEST PERIMETER? (S.)

WHICH SHAPES HAVE PERIMETERS THAT APPEAR TO BE THE SAME LENGTH? (R and T.)

WHICH SHAPE HAS THE SHORTEST PERIMETER? (M.)

WE HAVE MEASURED THE PERIMETERS OF SEVERAL CLOSED CURVES. WHAT DOES THE PERIMETER TELL US ABOUT THE CLOSED CURVE? (The length all the way around.)

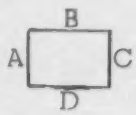


IS THE PERIMETER OF A CLOSED CURVE THE SAME AS THE CLOSED CURVE, OR IS IT ONLY ONE PROPERTY OF THE CLOSED CURVE? (It is only one property of the closed curve.)

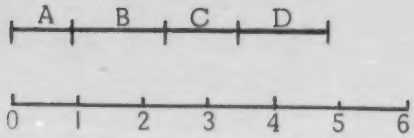
DOES THE CLOSED CURVE HAVE PROPERTIES OTHER THAN LENGTH? (Yes, it has shape.)

To reinforce the meaning of perimeter, draw a closed curve on the chalkboard and record its perimeter on a number line.

HERE IS A CLOSED CURVE.



HERE IS ITS PERIMETER STRETCHED OUT ALONGSIDE A NUMBER LINE.



DOES THE STRETCHED-OUT PERIMETER OF THE CLOSED CURVE LOOK THE SAME AS THE CLOSED CURVE? (No.) WHY NOT? (Because it isn't a closed curve anymore.)

Show the children a triangular object such as a property block. Ask them to measure the perimeter. If they measure each of the sides and then add the lengths together, ask if they can think of another, perhaps simpler, way to do it. Guide them to the possibility of putting a piece of string or yarn around the boundary, marking its length, and measuring the length of that portion of the string. You or the children may prefer to use narrow strips of paper instead of string. The children may wish to cut off the paper strip instead of marking it with pencil or finger. Have them apply this method to several objects and then introduce a circular object or some other object with a curved boundary. The children should see that the string or paper method will work for this also.

Let the children measure a number of objects found in the room and record the name of the object and its perimeter. Some of the following shapes would be appropriate: different sizes of paper, a book cover, desk top, tiles and shapes drawn on chalkboard or floor. Hold up different objects and elicit answers from the children about the perimeters.

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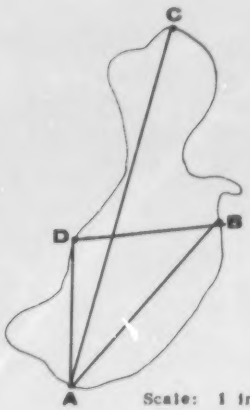
Activity D

Worksheet 13 may be used now or later. This worksheet involves further additional length measurements and introduces the idea that a map is drawn to scale.

Worksheet 13
Unit 12 Name_____

MAP OF BOTTLENECK LAKE



Scale: 1 inch = 1 mile

Worksheet 13, continued
Unit 12 Name_____

A map of Bottleneck Lake is drawn on the opposite page. One inch on the map stands for one mile on the real lake. There are four scout camps located at points A, B, C, and D along the shore. The scouts travel from one camp to the next by canoe. Find how far it is on the lake by measuring lengths on the map.

Scale: 1 inch stands for 1 mile

Distance BD = 2 miles

Distance AD = 2 miles

Distance DA = 2 miles

Distance BA = 3 miles

Distance CA = 5 miles

Two scouts in a canoe went from Camp B to Camp D, then to Camp A, and then to Camp C. How far was the whole trip?

2 + 2 + 5 = 9 miles

SUPPLEMENTARY ACTIVITIES FOR SECTION I

These activities may be used for review of length measurement whenever you think appropriate.

Activity A: Does It Fit?

The children may enjoy testing their ability to estimate the relative length of an object and the distance between two objects. Select two objects and ask questions about the space between them, e.g.: Would this desk fit between the wall and the bookcase? Can you guess? How can we find out? If the children suggest moving the desk, explain that the idea is good, but difficult, and that you want them to think of an easy, indirect way to compare the lengths. Guide them to measuring the desk and the space, and comparing the two. Let them use any reference unit they wish (handspan, armspan, string, set of books, etc.) for the indirect measurement.

You can vary this by asking: What object would fit between these lines (e.g., edges of a floor tile)? After the children make predictions, have them check with both direct and indirect comparisons.

Activity B: Making A Ruler

To make their own rulers or tape measures, have the children choose any unit of length which can be repeatedly marked on a stick, strip of paper, adding machine tape, ribbon, etc. Ask each child to find some interesting object, guess how long it is in his units, and then measure it. Let the children measure whatever they wish, and encourage imagination in the choice of units for the rulers.

Activity C: Even Steven

This game is obtainable at many toy departments or from Kohner Bros., Inc. (#119), 1 Paul Kohner Place, East Paterson, New Jersey 07407. This solitaire puzzle consists of eight sleeves and eight pegs. The sleeves are of various depths and the pegs of various lengths. Object of the game is to combine pegs and sleeves in such a way that all are of the same height when combined (one peg in one sleeve). Starting out with a jumbled assortment, the player counts the number

of moves it takes him to make the combined pegs and sleeves all even (level). An astute player may discover that he can first order all the pegs according to length, and then order the sleeves by looking at their bases to judge their depth. He can then confidently place the longest peg in the sleeve with the deepest hole and the next longest in the hole that is slightly shorter, etc., and solve the game quite quickly.

SECTION 2 MEASURING AREA

PURPOSE

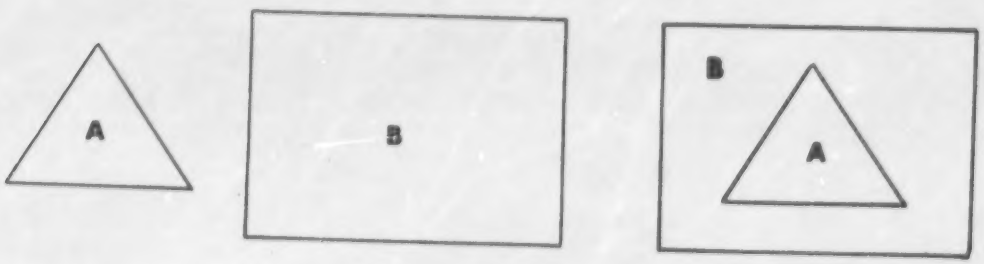
After completing the lessons in this section, a child should be able to:

- Compare the areas of two regions on flat surfaces directly by superposition, or indirectly with reference objects.
- Measure the area of a given flat region in terms of some area unit.
- Use several sizes of standard units in area measurement.
- Distinguish area from length and perimeter.
- Understand the invariance of most area comparisons.

COMMENTARY

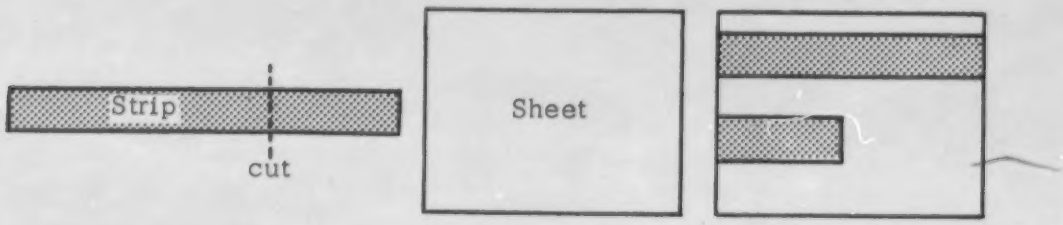
In Kindergarten Unit 5, Introducing Measurement, area was introduced as the size of a region on a surface bounded by a simple closed curve. The children were given a number of activities in comparing areas by placing one object on top of the other.

Here again they will determine area measurement by the covering process to find out how much room there is on a surface region. The area units they will use are not derived from units of length; that is, the children will not try to calculate areas from measurements of lengths. Instead, areas are compared by superposition. If surface region A can be placed entirely on region B with some of the surface still uncovered on B, then B has the greater area.

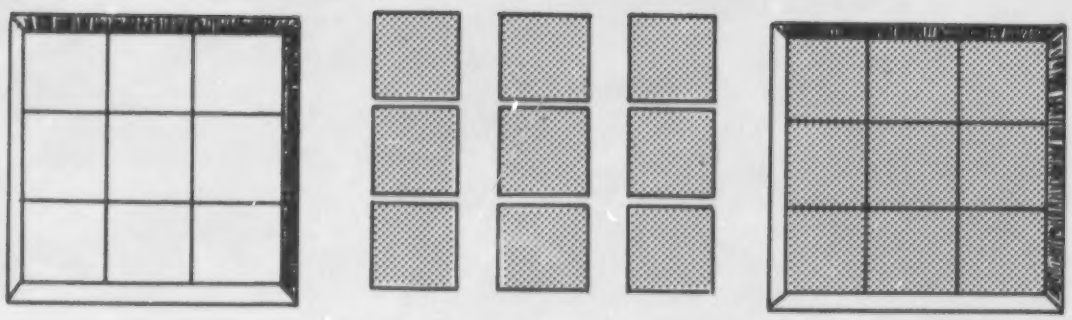


Area B > Area A

Under many operations, such as moving objects from one location to another without stretching them, the area relationship remains unchanged and the result of area comparison is then said to be "invariant." For example, rearranging the pieces of a puzzle or cutting up a piece of construction paper leaves the total area of the top surfaces unchanged. The children will use this concept of invariance (conservation) in making area comparisons and numerical measurements.



Area of Sheet > Area of Strip

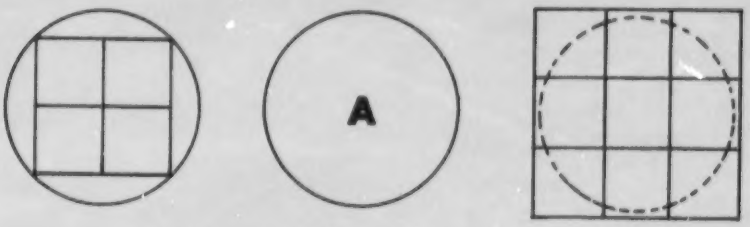


Tray 9 Squares Area of Tray ≐ Area of 9 Squares

From direct area comparison by superposing (including cutting and rearranging) the children go to indirect comparison with reference units. This leads to quantitative measurement: specifying a number or numbers and the unit used.

Objects that just cover each other can be said to have essentially the same area. By choosing the area of one such object as an area reference unit, measurements can be made by

applying the reference unit repeatedly to cover a region or by covering the region with replicas of the reference unit. Bracketing of an area between whole numbers of reference units is then possible, but more difficult than the similar operation in length measurement. The intervals may have to be relatively large.



4 squares < Area A < 9 squares

By using identical buttons, small erasers, paper clips, and other objects (lima beans are excellent) the children learn about standard reference units for measuring area. In Lesson 12, they use two transparent grids, one divided into square inches and another into square centimeters. Using both units emphasizes that there is more than one common standard reference unit and that precision depends on the size of the reference unit chosen.

Additional activities are suggested on page 86. These may be used where needed for reinforcing the various lessons, for review, or for individual children to do with a minimum of supervision.

Many of the lessons in this section contain more activities than can be done in one class period. If you find that all the activities are not necessary to develop the point of the lesson for your class, feel free to omit some. On the other hand, devote several class periods to a lesson wherever necessary.

Lesson 8: SUPERPOSING AND CUTTING

This lesson develops the concepts of area and of area comparison. The area of a region on a surface is determined by what covers it. Two areas can be compared by superposing, by placing one over the other.

Instead of learning formal definitions in this lesson, the children play with games that make them aware of area as a property by which to order or compare objects. Cutout puzzles, jigsaw puzzles, the Pentomino game (Activity A) and the tangram puzzle provided for Lesson 9 are helpful for the purposes of the lessons. These puzzles and games should be available for free play even before Lesson 8 is started.

Some excellent and inexpensive puzzles are available commercially. The tangram puzzle is sold as "Pythagoras" and the Pentomino game can be purchased as "Hexed." Other puzzles games may be obtained under such names as "Euclid," "Tormentor," "Voodoo," "Hi Jinks," "Sweetheart," and "Krazy Quilt." "Hexed" and "Krazy Quilt" are very difficult.

MATERIALS

-- for the class --

- any puzzles requiring a comparison or matching of areas
- property or parquetry blocks

-- for each child --

- Pentomino Game and tangram puzzle (provided in the back of the Student Manuals and in the back of this manual)
- Worksheet 14
- 2 sheets of 9" x 12" construction paper, different colors for different children

PROCEDURE

Activity A

Let the children play with whatever jigsaw, map, or cutout puzzles are available. They may do this individually or in pairs.

Tangram puzzles, Pentomino and similar games should be used now and made available for free play throughout the work in this section.

The standard game of Pentomino is played by two players. They sit across the empty board with the twelve pentominos spread on the table beside it. The first player takes one of the pieces and places it so that it covers any five squares of the board. The second player similarly places one of the remaining eleven pieces to cover five of the remaining empty squares. The play alternates until one player is unable to move, either because no remaining piece will fit or because no pieces are left. The player who cannot play is the loser. Some children enjoy playing the game alone. A simpler version can be played with dominoes instead of pentominos.

Activity B

Have all of the children in the class (or at least a dozen) crowd together on a cleared region of the floor. Draw a closed curve in chalk around them. Ask whether all can stand on the region inside the boundary you have drawn. (Yes.)

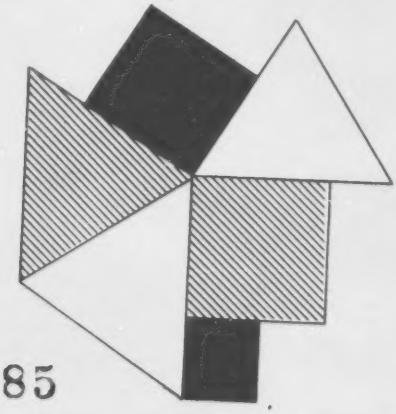
With chalk, mark off a smaller region on the floor and ask all the children to stand on the region inside this boundary.

WHY CAN'T YOU ALL STAND IN THIS REGION? (There is not enough room or space inside the closed curve.)

WHEN YOU THINK ABOUT HOW MUCH ROOM OR SPACE THERE IS IN A REGION, YOU ARE THINKING ABOUT AREA.

Activity C

Let the children play with blocks that can be shoved together to make interesting shapes. Property blocks with straight sides or parquetry blocks are appropriate.



Now have the children assemble on Worksheet 14 a shape made from blocks and trace closely around the outer edge. When the children have completed the worksheet they may wish to color the design they have made.

Worksheet 14
Unit 12 Name_____

Trace around a shape made from blocks.

Take the blocks away.

Can you fit them back in to fill the region?

Activity D

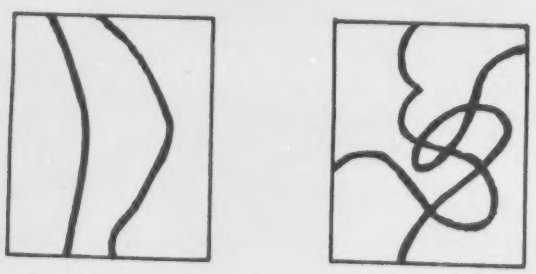
Give each child two sheets of construction paper of the same color, but try to see that neighboring children have different colors. Then ask that each child put one of his sheets on top of the other.

DOES ONE OF YOUR SHEETS OF PAPER HAVE MORE AREA THAN THE OTHER? (Both appear to be the same.)

Now have the children draw two curves on one sheet with pencil or crayon:

ON ONE SHEET, START AT THE BOUNDARY OF THE PAPER AND DRAW ANY CURVE YOU WISH THAT ENDS ON THE BOUNDARY.

START AGAIN AT THE BOUNDARY AND DRAW ANY CURVE THAT ENDS ON THE BOUNDARY OF THE PAPER.



Examples of curves children might draw

These curves may be open or closed, simple or non-simple, and may or may not cross each other.

Have each child cut his sheet of paper along the lines he has drawn. This will produce three or more (perhaps many more) pieces. Using them as a puzzle, have the child try to reassemble the pieces on top of the other sheet. Have him compare the areas.

DO ALL THE PIECES TOGETHER HAVE THE SAME AREA AS THE OTHER SHEET? (Yes, they appear to have the same area.)

This point should become clear: as nearly as they can tell, the area has not been changed by cutting up the sheet.

Now have each child compare the area of one piece with the uncut sheet.

WHICH HAS THE GREATER AREA — ONE OF THE PIECES OR THE UNCUT SHEET? (The whole sheet.)

HOW CAN YOU CHECK THIS? (By putting the piece on the uncut sheet; by the superposition test.)

Have each child select two pieces of his puzzle.

WHICH PIECE HAS THE GREATER AREA?

HOW CAN YOU CHECK THIS? (By putting one piece on the other.)

Many children will choose pieces for which superposition will work; that is, one piece can be placed upon the other and not project beyond it.

However, some children are likely to choose two pieces for which the test will not work. Each piece will project beyond the other, no matter how they are placed. The child may say, "Both of the pieces have more area," or "I can tell that this has the greater area just by looking."

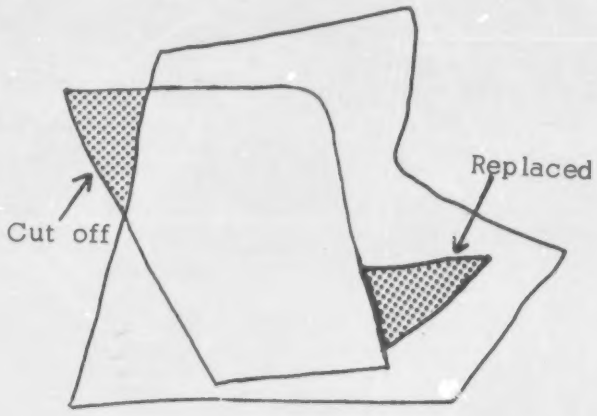
Select a pair of pieces such that one will require little cutting to make it fit on the other.

WHICH OF THESE PIECES HAS THE GREATER LENGTH?

WHICH HAS THE GREATER AREA?

CAN ANYONE SUGGEST A WAY TO CHECK OUR GUESS?

When the children discover that the superposition test does not work, guide them to suggest that a piece can be cut off one region so that the parts of that region can be made to fit on the other region.



Every child should have an opportunity to make an area comparison which requires cutting. The children may trade some complete puzzles with their neighbors.

For review, and for contrast, pick three pieces from different children and ask the class to arrange them in order by length, and also by area.

NOTE: Supplementary Activities A and D (page 86) are appropriate here.

Lesson 9: DIRECT AND INDIRECT COMPARISONS

This lesson leads children to perceive area relationships directly. They also use arbitrary standard units for comparing areas indirectly.

It is more difficult to compare relative areas visually than to perceive relative lengths. Children receive experience in using areas to solve problems in this lesson through the use of the old Chinese tangram puzzle.

MATERIALS

-- for the class --

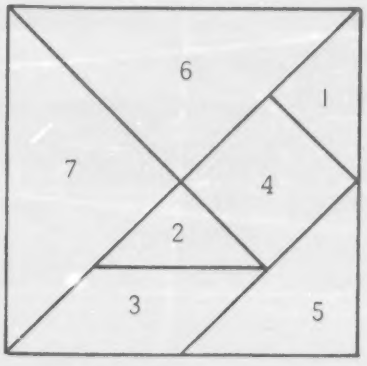
- plain sheets of paper

-- for each child --

- tangram puzzle (provided in the back of the Student Manuals and in the back of this manual)
- scissors
- Worksheet 15
- many identical objects such as lima beans, buttons, paper clips, or small blocks (for measuring area)

PREPARATION

When the tangram puzzle is cut along the heavy black lines and arranged properly, the pieces will produce an 8" x 8" square. The puzzle can also be cut from colored cardboard or from graph paper. You may want to make a copy for the flannel board.



PROCEDURE

Activity A

Ask the children to cut out the tangram puzzle, if they have not already done so.

Putting all seven pieces of the tangram puzzle together to form a square is a challenging problem for children of all grades. They can discover many other interesting shapes during free play, so make the puzzle available to the class for several days. Then ask questions about it and have the children check each answer by the superposition test. Instead of cutting a large piece, ask them to find two smaller pieces and reassemble these to match the greater area. By testing, they should discover that pieces 1 and 2 together have the same area as pieces 3, or 4, or 5.

Begin the questions by holding up pieces 1 and 2, the small triangles, and asking the children to name the shape. Then ask:

WHICH PIECE HAS THE GREATER AREA? (The area of the two triangles appears to be the same.)

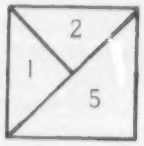
HOW CAN YOU CHECK THIS? (By placing one on top of the other.)

CAN WE MAKE A SQUARE FROM THESE TWO TRIANGLES? HOW? Have a child make a square.

CAN WE MAKE A LARGER TRIANGLE FROM THESE TWO TRIANGLES? SHOW ME HOW. Have a different child make a larger triangle.

Have a child place piece 5 with pieces 1 and 2.

NOW SHOW ME WHETHER OR NOT YOU CAN MAKE A A SQUARE FROM THESE THREE PIECES.



NOW SEE IF YOU CAN MAKE ALL THREE INTO ANOTHER TRIANGLE. (No, you cannot.)

COMPARE THE AREA OF PIECE 4, THE SQUARE, WITH THE AREAS OF THE SQUARES YOU MADE FROM PIECES 1 AND 2, AND FROM 1, 2, AND 5. WHAT DID YOU FIND OUT? (Pieces 1 and 2 together have the same area as piece 4. Pieces 1, 2 and 5 together have greater area than piece 4. Pieces 1, 2 and 5 together have twice the area of piece 4.)

CAN YOU MAKE A SQUARE AREA FROM PIECES 6 AND 7, THE TWO LARGE TRIANGLES? SHOW HOW.

HOW DOES THE AREA OF THIS SQUARE COMPARE WITH THE AREAS OF THE OTHER SQUARES YOU MADE?

CAN YOU ARRANGE SQUARE 4, THE SQUARE MADE FROM 1, 2 AND 5, AND THE SQUARE MADE FROM 6 AND 7 IN ORDER BY AREA?

HOW DO THEIR AREAS COMPARE? (Each larger one has twice the area of the one before it.)

Save the tangram pieces for use in other activities.

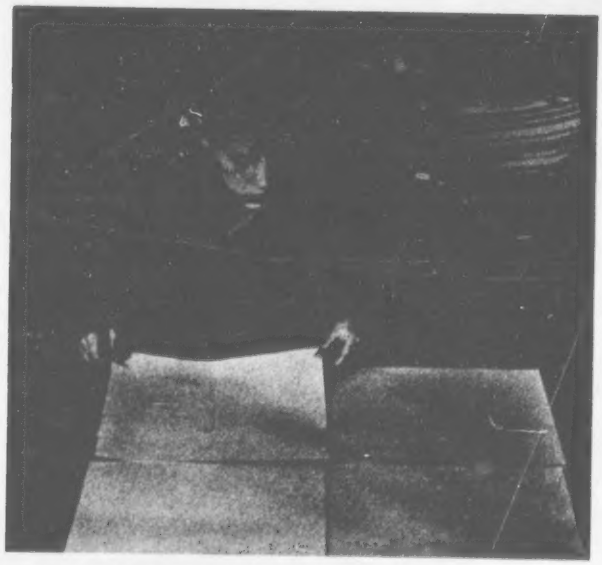
Activity B

Here the children make indirect comparisons with arbitrary standard units of their own choosing. Have on hand a number of paper sheets of uniform size and also quantities of lima beans, small erasers, paper clips or whatever small objects you selected from the materials list for the children to use as standard units for measuring area.

Begin by asking which has the greater area, the top of Johnny's desk or the top of your own desk. When the children name your desk top as the greater in area, ask:

HOW CAN YOU CHECK TO FIND OUT WHICH DESK HAS THE GREATER TOP AREA WITHOUT MOVING EITHER DESK?

If the children can't think of any way to do this, recall how they compared areas on the floor by covering the areas with children (in Lesson 8, Activity B). This should stimulate someone to suggest that a paper replica of one desk-top



region be made and then moved to the other desk. Or someone may suggest using a reference unit for comparing the areas. For example, a child's desk might be shown to have an area between that of four sheets and five sheets of paper. These papers can then be moved to your desk which has an area greater than all the sheets of paper.

Area of 6 sheets > Area of Johnny's desk

Area of 6 sheets < Area of teacher's desk

Now bring up the quantitative measuring of an area with reference units.

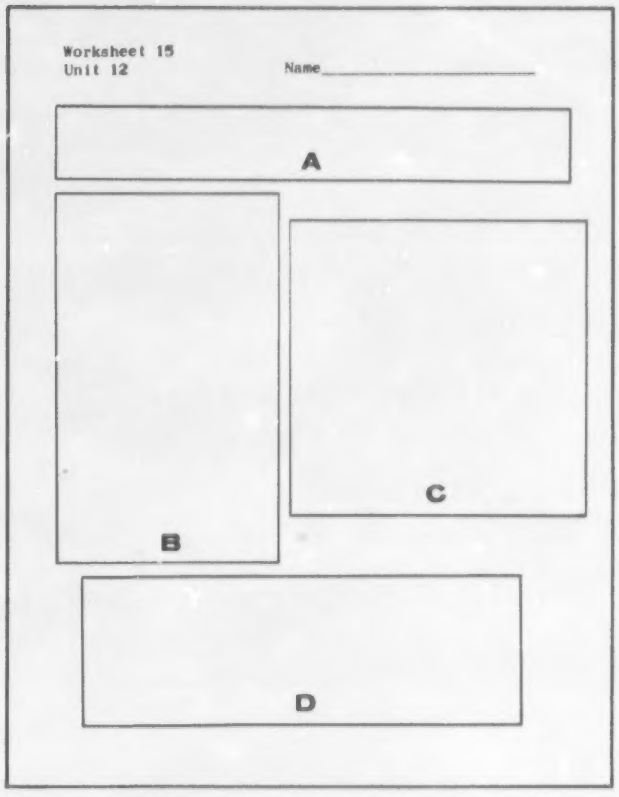
WE USED PAPER CLIPS TO MEASURE LENGTH. WHAT COULD WE USE TO MEASURE THE AREAS OF THE TANGRAM PUZZLE PIECES?

After the children have made their suggestions, bring out the sets of small objects (lima beans, little erasers, etc.). Be certain there are enough objects in each set to cover any area the children might wish to measure. Have each child select one tangram piece and one set of objects to use as his reference unit.

When the children select a second tangram piece to measure, they could cover the second shape by using the same set of objects as in the first region. By this method they can determine whether the second region requires fewer, the same number, or more units to cover than the first.

Activity C

On Worksheet 15, have the children measure the area of one of the shapes with the reference units from Activity B. Arrange to have different children measure different shapes. Record the results of their measurements on the chalkboard. From this data, ask the children to decide how to order the shapes by area. They should see that comparison is difficult because various reference units were used, and because some objects fit together more closely than others.



Save Worksheet 15 for use in the next lesson.

Lesson 10: MEASURING AREA WITH COMMON STANDARD UNITS

The children use square inches and square centimeters as common units of measurement which makes communication of the results of their area measurements easy. As a convenient means of associating area measurement with the number line, the face of the black Minnebar is introduced as a unit of area measurement (actually one-fourth square inch). If a region can be exactly covered by Minnebars and those Minnebars are lined up beside a number line with half-inch spacing, the number of area units can be read directly from the number line. The same method can be used if you cut a shape (or pattern of it) into strips one inch wide and place the strips end to end beside a number line of inches.

MATERIALS

-- for each child --

- 15 one-inch square counters (or squares 1" x 1" cut from 3" x 5" filing cards)
- Minnebars
- piece of tangram puzzle from Lesson 9 or large, thin property block
- shapes A, B, C, D from Worksheet 15 (from Lesson 9)
- rulers

PROCEDURE

Activity A

Have the children turn to Worksheet 15 and recall the problem that arose when they measured the shapes with a variety of units: they could not make satisfactory comparisons.

HOW COULD WE MAKE COMPARING THE AREAS EASY?
(By all using the same units for measuring the area.)

Provide each child with 15 one-inch counters and a ruler. Ask them to measure the counters using the inch side of their rulers.

WHAT IS THE LENGTH AND WIDTH OF THESE UNITS OF AREA? (One inch by one inch.)

Tell the children that this unit is known as a square inch. Have each child measure the area of each of the shapes by placing the units to cover the region exactly. The results can be recorded by writing "Area of A $\doteq$ 7 square inches" on the shape. Now have the children order the shapes by area and record the order:

Area of A < Area of D < Area of B < Area of C.

Assign partners to the children or have them choose them. Ask them to solve the problem of constructing a shape that has five units on each side and then to tell how many units are needed to show the area covered by this shape. After the shapes have been constructed and the area units counted, the children can measure the perimeter with their rulers. They can then discuss the difference in the two measurements — area and perimeter. (The children are not expected to find a correlation.)

Activity B

Give one piece of a tangram puzzle or a thin property block to each child, selecting the larger pieces for children who can handle large numbers easily.

HOW CAN YOU MEASURE THE AREA OF THE TANGRAM PIECE? (We could cover it with some objects. We could count the squares on the tangram piece.)

COULD YOU USE MINNEBARS TO MEASURE THE AREA? HOW? (Cover the region with Minnebars and then count the Minnebar units.)

HOW COULD YOU MAKE THE COUNTING EASIER?

Suggest that the Minnebars can be counted more easily if they are removed from the region and rearranged in some orderly fashion. Some children may suggest that the Minnebars could be counted by matching them against ten bars, etc.

Some children will have put the Minnebars in rectangular arrays. Point this out and ask how long (in Minnebar units)

each row of bars is, how many rows there are, and what the measure of the area is in Minnebar units. This provides some background for pre-multiplication ideas, which are introduced in Unit 13.

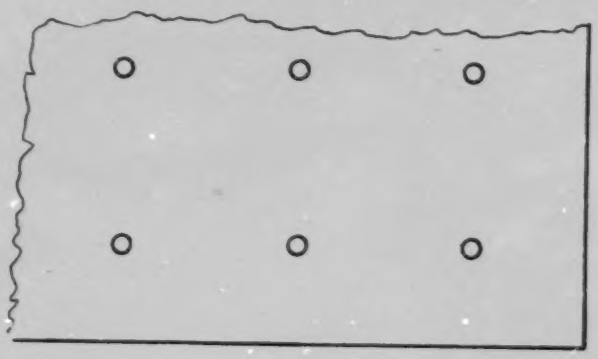
Some children may wish to use the paper piece of the puzzle itself instead of the Minnebars. They can do this by cutting a piece into strips all of the same width, carefully following the lines ruled in one direction across the piece. The strips can be put end to end in one long row and measured with the number line. For the triangular pieces, this may lead to the combining of half units into wholes, an idea which will be developed in Lesson 12.

NOTE: Supplementary Activities B and C (page 86) are especially useful here.

Lesson 11: MEASURING AREA ON A PEGBOARD

The distinction between a simple closed curve (the perimeter) and the region it encloses on a flat surface (the area) is reinforced in this lesson by having the children use pegs to fill the regions of closed curves on pegboards. The surface within such a closed curve is approximately measured by counting the number of pegs that can be placed within it. The children will discover that even though the perimeter of a closed curve remains the same in length, the area it encloses will be greater or less depending on the shape of the closed curve.

The activities of this lesson are designed for a pegboard twelve by twelve inches, with holes one-eighth inch in diameter spaced in a rectangular array, and with the centers of the holes one inch apart.



Section of standard pegboard showing arrangement of holes with centers 1" apart.

MATERIALS

-- for each group of four --

- 12" x 12" pegboard
- 40 to 50 golf tees, preferably all of the same color within a group
- plastic clothesline, between 21" and 22" long
- masking tape, about 2"

PREPARATION

For each group of four children, cut a length of plastic clothesline that is between 21 and 22 inches long. Then prepare a pegboard for each group in this fashion: turn the pegboard

face down and insert a golf tee into each corner hole. This will hold the board high enough above the desk so that the children can push golf tees firmly into the face of the pegboard.

PROCEDURE

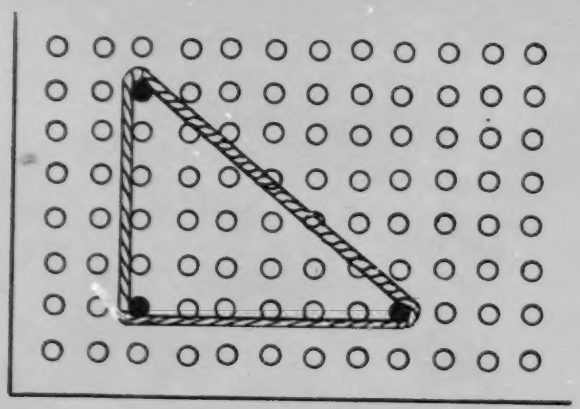
Activity A

Distribute a pegboard, tees, a piece of clothesline and some masking tape to each group of four children. Ask them to measure the length of the rope in inches, with rulers, and record the result.

L of rope > 21 inches

L of rope < 22 inches

Help them tape the ends of the rope to form a closed curve. Then stretch it tightly around three pegs placed in the shape of a triangle on the pegboard.



Discuss this arrangement with the class.

WHAT HAVE WE FORMED? (A simple closed curve, a triangle.)

WHAT IS THE PERIMETER OF THE TRIANGLE? (It is the length of the clothesline – between 21 and 22 inches long.)

HOW COULD WE FIND THE AREA OF THIS TRIANGLE?

Commend appropriate suggestions and have the children test them. Try to elicit the idea of using pegs or golf tees in the holes inside the curve. If no child suggests it, show the tees.

COULD WE USE THESE GOLF TEES IN SOME WAY? HOW? (By putting one tee in each hole inside the curve and then counting all the tees.)

Have each group measure the area of its triangle. Repeat the process with the loop formed in the shape of a square or other rectangle, or any other simple closed curve.

Activity B

Let two groups work together. Provide each of the two groups with golf tees of a different color. Then ask each group to make a simple, closed curve of any shape on the pegboard and fill it with the tees.

WHAT IS THE PERIMETER OF EACH SHAPE? (Between 21 and 22 inches.)

WHAT MEASURES THE AREA OF EACH REGION INSIDE A CLOSED CURVE? (The number of tees it takes to fill it.)

HOW CAN WE COMPARE THE AREAS INSIDE THESE DIFFERENTLY-SHAPED, CLOSED CURVES?

Let each pair of groups try various methods of comparing their areas within curves of different shape (but the same perimeter). Some possible methods are suggested:

(1) The children can compare the number of tees used by moving those from within one curve to within the other. The result will be that there will be too few tees, too many, or just enough.

(2) Tees that measured each curve can be arranged in separate, but similar, arrays or in two straight lines to make them easier to count. Straight lines may require either a larger pegboard or the lining up of several pegboards side by side. Arrays with ten in each row are convenient.

Conclude the lesson by bringing up these questions:

> WHEN THE SAME LOOP IS MADE INTO A TRIANGLE, A SQUARE, A LONG RECTANGLE OR A NEARLY-CIRCULAR SHAPE, IS THERE MORE ROPE IN ANY ONE? A GREATER PERIMETER? (No, all have the same perimeter.)
>
> IN WHAT SHAPES IS THE AREA GREATEST? IS IT GREATEST WHEN THE LOOP IS MADE INTO A TRIANGLE, SQUARE, LONG RECTANGLE, OR APPROXIMATE CIRCLE?

Let the children discover through their own experiments that the circle has the greatest area for a given perimeter.

Lesson 12: USING GRIDS TO MEASURE AREA

While receiving experience in the use of more than one common standard unit of measurement, the children develop new rules for measuring, so that their results will be more accurate. They also try new methods of measuring area. What they learn explicitly is that area measurement of a given shape is usually the same, regardless of the position of the shape on a flat surface: that area is usually invariant with change of position.

The children use transparent (tracing paper) grids and graph papers marked off in square inches or centimeters to cover regions with reference units. They will find these easier to use than the loose, separate squares and that the grid papers make counting the units simple.

By using the square centimeter unit, which is introduced simply as another convenient reference unit, the children may discover that they get more precise results because of the smaller size. The accuracy of the measurements is also increased by the combination of two half-units to give a whole unit.

MATERIALS

-- for each child --

- 15 square counters (1" x 1")
- transparent grid of 1" x 1" squares (provided in the back of the Student Manuals and in the back of this manual)
- transparent grid of 1 cm x 1 cm squares (also provided in the back of all manuals)
- graph paper (1" x 1" squares)
- graph paper (1 cm x 1 cm squares)
- string
- Worksheets 16 through 18

PROCEDURE

Activity A

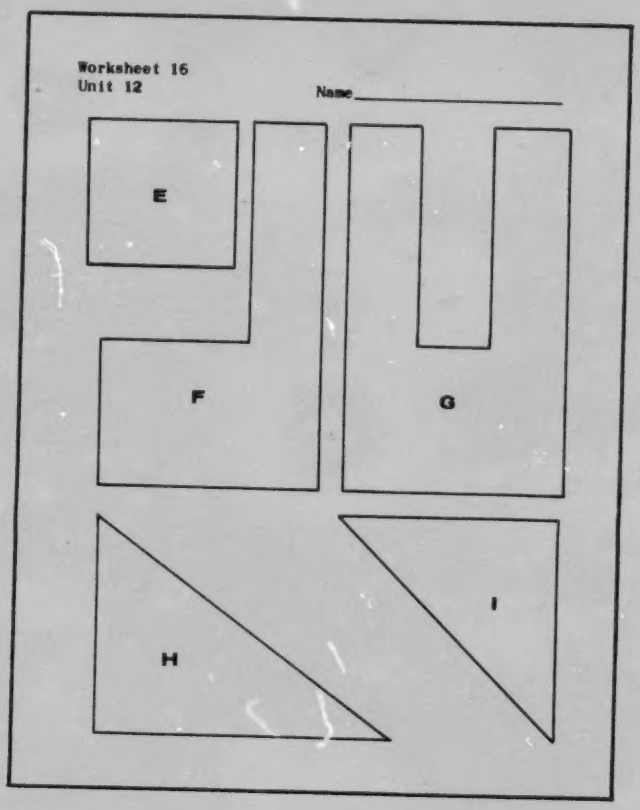
Give the children each 15 (1" x 1") counters and have them

use these to measure the areas of the shapes on Worksheet 16. Then ask them to turn to the back part of their Student Manuals and find the transparent sheet of one-inch squares. Ask them to tear this sheet out carefully. Explain that a paper with markings like this is called a "grid sheet."

WHAT DO YOU NOTICE ABOUT THIS GRID SHEET? (We can see through it. It has squares marked on it. The squares seem to be of the same size as our counters.)

HOW CAN YOU CHECK TO SEE IF THE SQUARES ON THE GRID ARE THE SAME SIZE AS THE CC JNTERS? (By placing the counters on the grid.)

CAN ANYONE SEE HOW WE COULD USE THIS TRANSPARENT GRID SHEET TO MEASURE THE AREA OF A SHAPE? (We could put a grid over the shape and count the number of grid squares it takes to cover the shape.)



You may wish to demonstrate by making a transparency and using an overhead projector. Then have the children try this method to measure the area of shape F. Then let them use it for measuring the areas of shapes E and G. Have them repeat the measurement of the area of G when G is placed in different positions on the desk and when the worksheet is turned over.

IS THERE ANY CHANGE IN THE AREA OF SHAPE G WHEN WE CHANGE ITS POSITION OR TURN IT OVER? DOES THE AREA OF SHAPE G VARY WITH ITS POSITION?

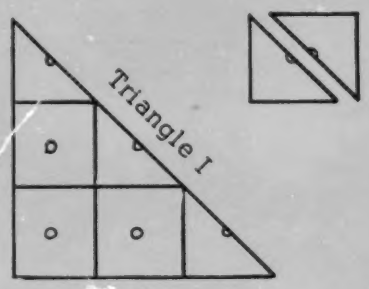
The children should see that the area stays the same (is invariant) regardless of the position of the shape.

Activity B

Have the children use the grid to measure the area of shape H on Worksheet 16. Now they certainly will ask questions about whether to count squares that fall only partly on the region. Find out what solutions the children can offer for handling the situation. Then suggest:

> THE DOTS IN THE CENTERS OF THE SQUARES COULD HELP US TO DECIDE. ONE RULE WE MIGHT USE WOULD BE TO COUNT A SQUARE IF ITS CENTER FALLS ON THE REGION.

Have the children try this rule on Triangle H and then on Triangle I. For Triangle I, the centers of three squares will fall just on the boundary, which will raise a new problem. Ask the children how many of the triangular pieces of the grid units it would take to make a square unit. They may see that two triangles make a square and that they can get more accurate area measurements by counting two of the triangles (half-squares) as a whole unit. Thus, Triangle I has an area between four and five square inches (actually four and a half). Counting whole squares only, the result is between three and six square inches.



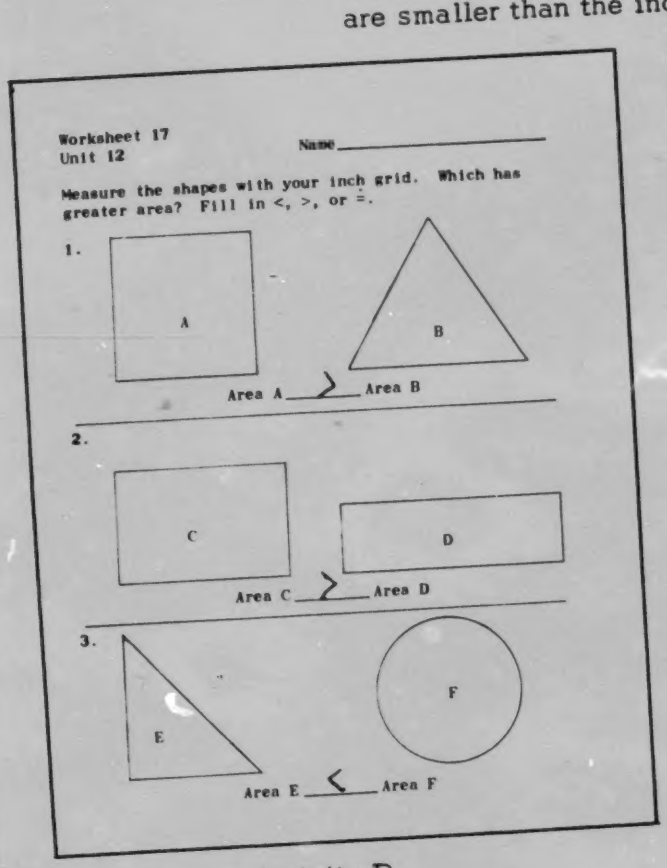
Activity C

Have the children repeat the measurements of shapes E through I, using the centimeter grid from the back of their Student Manuals. Introduce this grid as having area units that are one centimeter by one centimeter, and explain that the square centimeter is another common reference unit of area measurement. Let the children use whatever rule for counting units they feel is appropriate. Since the number of units involved will be quite large for some of the shapes, you may prefer to issue centimeter graph paper, and have the children cut out and place the shapes on the graph paper and trace around them.

They can check each unit as they count it or even number the squares consecutively (as a review of numeration and as a means of keeping track of the counting).

In the discussion, compare the number of units in the area of one shape when each kind of common unit of area is used (inch squares and centimeter squares).

WHY ARE THERE MORE CENTIMETER SQUARES? (They are smaller than the inch squares.)



Worksheet 17
Unit 12

Name _____

Measure the shapes with your inch grid. Which has greater area? Fill in <, >, or =.

1.

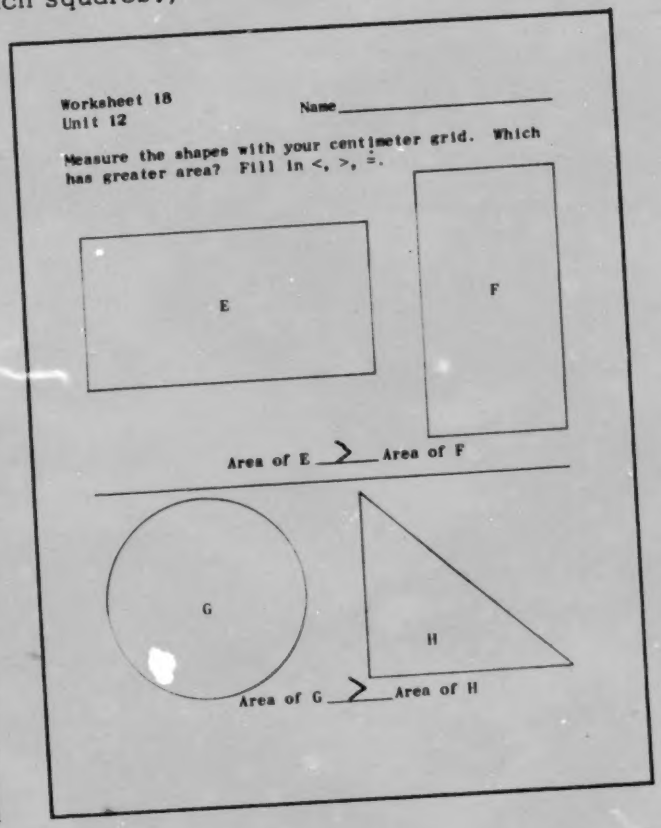
Area A ___ Area B

2.

Area C ___ Area D

3.

Area E ___ Area F



Worksheet 18
Unit 12

Name _____

Measure the shapes with your centimeter grid. Which has greater area? Fill in <, >, =.

Area of E ___ Area of F

Area of G ___ Area of H

Activity D

Have the children use their inch and centimeter grids to complete Worksheets 17 and 18. Help them with the directions and allow them to compare answers. You may wish to have them explain how they arrived at their conclusions.

NOTE: Ask each child to bring a frozen-juice can (6 oz. size) and a Jell-O box (3 oz. size) for Lesson 13.

SUPPLEMENTARY ACTIVITIES FOR SECTION 2

The following activities can be introduced whenever you feel it desirable to reinforce or review ideas developed in this section. They are appropriate after the lessons indicated.

Activity A (After Lesson 8)

Pentomino game.

Activity B (After Lesson 10)

Display various two-dimensional objects — Kleenex, paper toweling, paper napkins, etc. Ask children to measure and compare the areas of the samples.

Activity C (After Lesson 10)

Let the children devise their own standard units for measuring area when they do Supplementary Activity B, which requires area comparison.

Activity D (After any lesson in this section)

Provide a variety of natural objects and let the children devise methods for measuring and comparing their areas. Irregularly-shaped leaves and rocks will be challenging.

SECTION 3 MEASURING VOLUME

PURPOSE

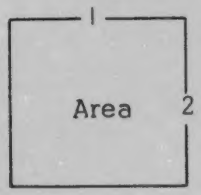
This section contains:

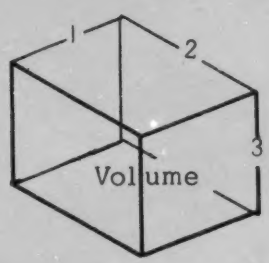
- A review and extension of the concept of volume.
- Activities in applying measurement concepts and techniques to three-dimensional regions.
- Practice in measuring volume by filling containers and by determining the amount of water displaced by a solid object.

COMMENTARY

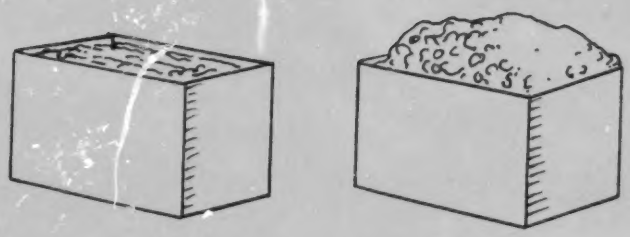
The presentation of volume here is like that of Unit 5, Introducing Measurement: volume is considered as the measure of the interior of an object — the measure of a spatial region within a simple closed surface.

In Section 2, the children studied area as the measure of the amount of surface in a flat (two-dimensional) regior within a closed curve. In this section they will be measuring a three-dimensional region.





Since volume is the measure of the region bounded by a closed surface, the volume of a rock is a measure of the material within the surface of the rock. The volume (capacity) of a box is a measure of the material that will fill the box. If the box has no cover, we "close" the box with an imaginary surface. This surface is flat when the box is filled level with the top, but curved when the material is heaped above the top. (See illustration on next page.)



Just as the area of a region or a surface was measured by finding how many reference objects of unit area would cover it, so the volume (capacity) of a container can be measured by finding how many reference objects of unit volume can be fitted within it.

One way to determine the volume of a solid, such as a rock, is to measure the amount of water it displaces in a container. This is done by marking the water level before the rock is put in the water and again after the rock is submerged. The measure of the volume of water between the two marks is a measure of the volume of the rock, since that is the amount of water displaced by the rock. In other words, the rock is now filling a region formerly occupied by the amount of water between the two marks.

The volume of the crockery material in a cup could be measured in this way, also. Obviously, this volume would not have to be the same as the volume of liquid which would fill the cup (its capacity). You may be familiar with the formulas for determining the volumes of simple solids — the volume of a rectangular box is its length times its breadth times its height or the area of its base times its height. However, it is not intended that these geometric relationships be developed here. At this point the children will simply be given experience in comparing the capacity of containers by filling them with some reference objects or a liquid, and finding the volume of some objects by water displacement.

The word "volume" is likely to be familiar to the children in a different sense — in connection with sound. Even there we could think of one guitar amplifier "filling more rooms" with sound than another.

In Lesson 13, the children will compare the volumes of a Jell-O box and a frozen juice can, using corks as the units of volume. In Lesson 14 the children are introduced to more conventional units of volume such as the quart, pint, etc. Lesson 15 leads them to find volumes by measuring the amount of water displaced by a submerged object — the unit of volume being a Minneglass (a small plastic container with a volume of about one ounce).

Lesson 13: USING A STANDARD VOLUME UNIT

This lesson extends the children's experience with quantification by having them measure the capacity of containers.

The children are guided to use an arbitrary standard (a cork) as a unit of volume measurement. They then use the standard corks to compare the capacity of a fruit juice can with that of a Jell-O box. The results are recorded in the form of histograms that show that, even with standard units, we may get variation in the results of measuring a quantity. This point is not emphasized, however.

This lesson is not divided into separate activities. The suggested questions are structured to give children an understanding of capacity, direct comparison, and indirect comparison.

(The histogram and worksheets should be saved for use in the next lesson.)

MATERIALS

-- for the class --

- 2 containers of slightly different volume, but markedly different shape, such as a 10- to 12-oz. soup can and an 8-oz. cottage cheese container
- enough sand or salt to fill the larger container
- histogram materials from Lesson 6, transparencies or charts

-- for each child --

- frozen juice can (6 fl. oz.) and Jell-O box (3 oz. size), to be brought by the children
- about 60 corks, size #4 (12-oz. container full)
- Worksheet 19

PROCEDURE

Hold up the soup can and the cheese container. Have the sand out of sight, but nearby.

WHICH OF THESE WOULD HOLD MORE ICE CREAM? IS THE CAPACITY OF THE CAN GREATER THAN, LESS THAN, OR ABOUT THE SAME AS THE CAPACITY OF THE CHEESE CONTAINER?

HOW COULD WE COMPARE THE CONTAINERS TO FIND OUT WHICH WOULD HOLD MORE?

Some children may be confident they can tell which container has the greater volume simply by looking at them side by side, but ask:

HOW CAN WE BE SURE?

CAN WE DECIDE BY FITTING ONE CONTAINER INSIDE THE OTHER? BY A DIRECT COMPARISON?

IS THERE SOME OTHER WAY OF COMPARING WHAT THEY WILL HOLD? BY AN INDIRECT COMPARISON?

If some child suggests that you fill the containers with ice cream to make the comparison, explain that it would be a good way but unfortunately you haven't any ice cream. Then bring out the sand.

COULD WE USE THIS SAND? HOW?

Let children demonstrate to the class that the comparison can be made by filling one container (level with the top) with the sand and then pouring the sand into the other.

Show the class a frozen juice can and a Jell-O box.

WHICH WOULD HOLD MORE ICE CREAM, THIS CAN OR THIS BOX?

HOW CAN WE COMPARE THEM TO FIND OUT? (We could fill one of them with sand and then pour the sand into the other.)

To bring up the idea of an indirect comparison of volume, use an imaginary situation like the one on the following page.

I KNOW A GIRL NAMED JEANNETTE WHO IS A FIRST-GRADER IN CANADA. SHE LIKES ICE CREAM, TOO. HER ICE CREAM COMES IN A BOX. IF WE COMPARED OUR BOX AND CAN, DO YOU THINK IT WOULD HELP US DECIDE HOW MUCH JEANNETTE'S CONTAINER WOULD HOLD? (No.)

WHY WOULDN'T IT? (Because we don't have one of Jeannette's ice cream boxes here. We don't know what kind of boxes they are.)

In the discussion, lead the children to recall that numbers helped when we compared lengths and areas.

WHAT COULD WE USE TO FILL OUR CONTAINERS AND THEN COUNT?

WOULD THE SAND BE GOOD TO USE? WOULD THE GRAINS OF SAND BE GOOD UNITS OF VOLUME? WHY NOT?

The children probably will see rather quickly that it would be impractical to try to count the grains of sand.

WHAT MIGHT WE USE AS UNITS OF VOLUME?

Suggestions to use peanuts in the shell, cranberries, or popcorn would be good. Elicit from the children that these objects could be used to fill the containers and the number in each container could be counted and compared with a similar count which Jeannette could make.

I DON'T KNOW WHETHER JEANNETTE COULD GET THE THINGS YOU SUGGESTED, BUT I KNOW THAT SHE HAS A LOT OF CORKS LIKE THESE. HOW COULD WE USE THEM FOR A COMPARISON?

Guide the discussion toward the following activity:

Divide the class into pairs and see that each pair has a juice can, a Jell-O box and about 60 corks (12 oz. container full). Have the children estimate which container will hold more corks and record their guess on the top of Worksheet 19. Then have them fill one container with corks and pour the corks into the other container, recording their results on the

(Answers will vary.)

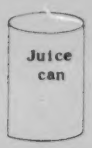
Worksheet 19
Unit 12 Name_____

Which container will hold more corks — the box or the can?

1. I think that the box will hold more corks than the can.

2. By pouring, I found that the can holds more corks than the box.

3. The box holds 23 corks.

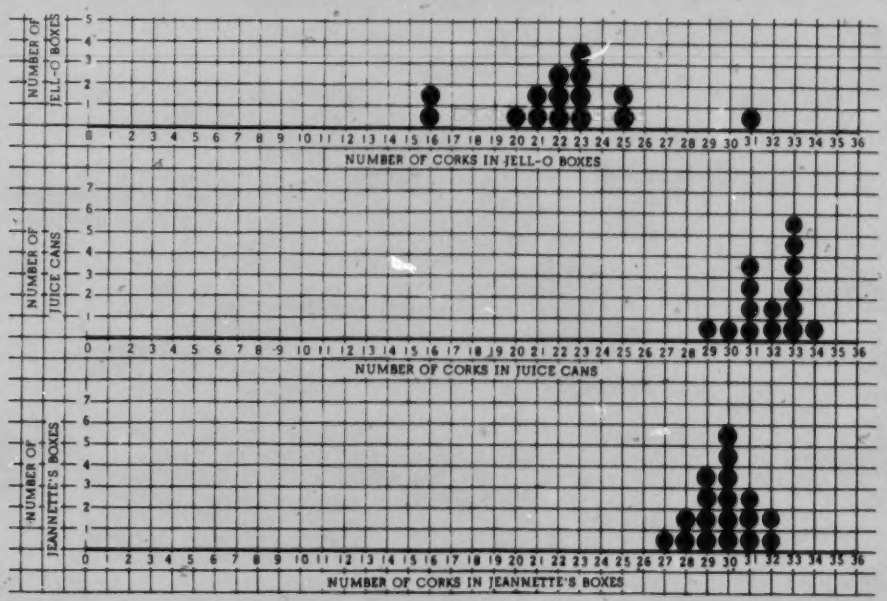


4. The can holds 33 corks.

5. Volume of box < Volume of can.
(Use >, <, or =.)

worksheet. Next ask them to determine by trial the number which each will hold. Point out that by "full" we will agree to mean level with the top, not heaped up. After they have made the trials and recorded the results, they are to complete the sentence at the bottom of the sheet.

To develop the idea that there is variation in such measurements and to further illustrate the use of a histogram, have the class record their results on two histograms. You may use the pegboards from Lesson 6, transparencies for an overhead projector or large sheets of paper for the histograms. Each pair should record its own data.


NUMBER OF JELL-O BOXES
NUMBER OF CORKS IN JELL-O BOXES
NUMBER OF JUICE CANS
NUMBER OF CORKS IN JUICE CANS
NUMBER OF JEANNETTE'S BOXES
NUMBER OF CORKS IN JEANNETTE'S BOXES

These histograms show the results of the measurements one class made.

Emphasize that there is no single "correct" result, only a "most likely" result. Give these reasons to help the children accept different results: the top of the container is not well defined; corks are packed in different ways; or corks and container vary slightly. Have each child repeat his measurements to illustrate the variation in packing. When the charts are complete and have been discussed, turn to Jeannette's chart and show it to the children.

THIS IS A COPY OF THE CHART THAT JEANNETTE'S CLASSMATES MADE FOR THEIR BOXES.

Explain that a portion of the number line was cut out so the chart could be made larger. There were no marks on the section that is missing.

WHAT DOES JEANNETTE'S CHART TELL US?

The children should see that the histograms indicate that Jeannette's class had boxes which would hold more ice cream than the Jell-O boxes but less than the juice cans.

IF EACH CHILD RECORDED A BOX ON THE CHART, HOW MANY CHILDREN ARE IN JEANNETTE'S CLASS?

Lesson 14: VOLUME OF A LIQUID AND INVARIANCE

The activities of this lesson are designed to:

- Give the children experience in comparing volumes of liquids.

First all the children will receive the opportunity to pour water into containers of various sizes and shapes. The activities lead children to realize that the shapes of the containers tend to confuse estimations of volume and making comparisons. They also learn that if liquid A has a greater volume than liquid B when compared in one container, liquid A will also have a greater volume when the liquids are compared in a different container. By definition or agreement, we assume that the volume of the liquid does not change when the liquid is poured from one container to another.

MATERIALS

-- for the class --

- 8 (1-oz.) Minneglasses
- 4 (8-oz.) plastic containers
- 4 (16-oz.) plastic containers
- 2 (32-oz.) plastic containers
- 1 (64-oz.) plastic container
- large container of water
- small amount of food coloring (optional)

-- for each group of four --

- tray
- 1 milk carton (1 qt.) of water
- 2 tall (12-oz.) plastic "glasses"
- 2 shallow (12-oz.) plastic "bowls"
- 4 (1-oz.) Minneglasses
- 2 (16-oz.) plastic containers

PREPARATION

Place on a tray the materials listed for each group of four.

PROCEDURE

Activity A

In some appropriate area of the classroom, place an assortment of 1-, 8-, 16-, 32-, and 64-ounce containers. Provide one large container of water. At convenient times, let one or two children "play" with the containers, pouring water from one container to another. The purpose is simply to give children the opportunity to discover relations between the volumes of the various containers.

Activity B

When all the children have had some experience with Activity A, select a tall, 12-ounce container and a shallow 12-ounce container and fill them with liquid (preferably colored water). Show the class these two containers and challenge the children with a problem similar to this:

> SUPPOSE THE LIQUID IN THESE TWO CONTAINERS IS VERY VALUABLE, AND THAT WE CAN KEEP ONLY ONE CONTAINER FULL. WHICH SHOULD WE KEEP, THIS ONE OR THAT ONE? WE WANT TO BE SURE TO KEEP THE ONE THAT HOLDS THE MOST.

Then say:

> I WILL GIVE YOU SOME CONTAINERS JUST LIKE THESE TWO, SOME WATER, AND SOME OTHER CONTAINERS, AND LET YOU EXPERIMENT TO SEE IF YOU CAN FIND WAYS WE CAN BE SURE.

Divide the class into groups of four and give each group a tray of materials.

Let each group develop its own methods of making the comparison. Some possible methods are for the children to:

1. Pour liquid from one container into one of different shape.

2. Fill both a tall and a shallow container, pour liquid from one into a second of the other shape, and then make the comparison.

3. Fill each type of container, pour from each into a separate pint container and make the comparison.

4. Fill each container by adding one Minneglass of water at a time and counting the number of glassfuls added.

When all groups have developed and tested several methods, let the children decide which container of the colored ("valuable") liquid they would keep. Discuss the various methods and the fact that the different shapes of the containers make it difficult to estimate which can hold more liquid, but that by pouring the liquid into containers of the same shape, we can find out which holds more (has greater volume).

NOTE: Some of the materials used in this lesson will be used again in Lessons 15 and 16. You may wish to look at the Materials Lists for those lessons (pages 98 and 100) and keep out the plastic containers, milk cartons, etc., specified in the lists.

Lesson 15: USING COMMON VOLUME UNITS

This lesson is planned to:

- Acquaint the child with the quart, pint, and cup as units of volume.
- Demonstrate the relationship between these common units of volume.

The children are challenged with another problem involving the relative volumes of a quart, pint, and cup. Working in small groups, they again develop and test methods for solving the problem. While the "1 quart = 2 pints = 4 cups" relation is not the main feature of the lesson, it can be developed as a review of set relations, counting, or addition.

MATERIALS

-- for each group of four --

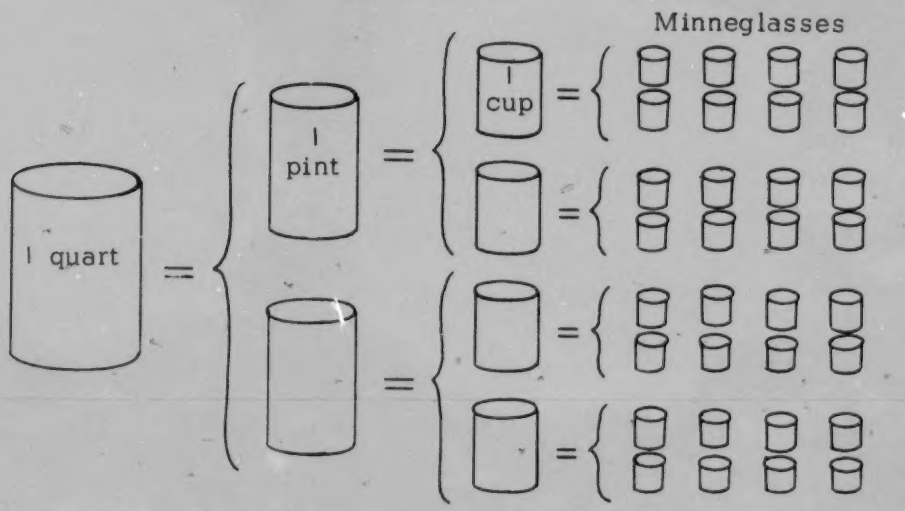
- 1 tray
- 2 (1 qt.) milk cartons (1 empty, 1 filled with water)
- 1 (16-oz.) pint container
- 1 (8-oz.) cup container
- 4 (1-oz.) Minneglasses

PROCEDURE

Show the children containers of quart, pint, and cup capacity. Present the class with the problem, perhaps in story form, of having to decide which would give them more of a delicious punch for a party — 2 quarts, 5 pints, or 9 cups. On the chalkboard, or by a display of the appropriate number of containers of each size, indicate these choices of amounts and the units involved.

Divide the children into groups of four and challenge each group to plan a way of deciding which choice of containers would give the most punch. Distribute trays of materials and allow time for the development and trial of various methods.

In the class discussion of the results, watch for cases where the numerical relationships between quart, pint, cup, and Minneglass are discovered. These relationships could be displayed by having the children make the appropriate arrangement of containers. At this point, label the larger containers: quart, pint, cup. Use set terminology, e.g., "A set of four cups full of liquid would fill a set of two pints or a set of one quart," in discussing the various displays, one of which could look like this:



1 quart = 2 pints = 4 cups = 32 oz.

1 pint = 2 cups = 16 oz.

1 cup = 8 oz.

(Omit the Minneglass relation unless it is brought up by your children or you feel that they can benefit from this review of addition.)

Lesson 16: MEASURING VOLUME BY WATER DISPLACEMENT

The plan of this lesson is to have children discover how to use displacement of water as a method of measuring volume.

The activities probably work best when done in two days. On the first day, Activities A, B and C demonstrate the invariance of displacement with change of the shape of an object and the displacement of water in overflow and non-overflow conditions. On the second day, Activities D, E and F review the concept of displacement by using marbles, marking calibrations on a cylindrical container, and having the children do individual experiments. All experiments should be performed on a level surface.

MATERIALS

- tray
- 12-oz. plastic container (tall, cylindrical, uncalibrated)
- saucer
- 2 Minneglasses
- piece of plasticine shaped to fit a Minneglass
- thin plastic, such as Saran Wrap
- thread or string
- several rubber bands
- felt tip marking pen
- sponge or paper towels

-- for each pair --

- 1 tray
- 12-oz. container of water
- 12-oz. container (tall, cylindrical, uncalibrated)
- 1 Minneglass
- plasticine (about 2 Minneglasses full)
- small objects suitable for use in containers of water
- paper towels

PREPARATION

To shape plasticine for your demonstration, line a Minneglass with a thin plastic and pack the glass just full of plasticine. Remove the plasticine from the glass and the plastic, then tie a thread or thin string around it to make it easier to use in water. An alternate method would be to omit the plastic wrap and form the plasticine directly in the Minneglass. The Minneglass must then be cracked to remove the shaped plasticine.

PROCEDURE

Activity A (Demonstration)

Depending on whether the children have had Unit 5, this activity is either a review or an introduction to the idea that the displacement of water can show that the volume of a solid is conserved when its shape is changed or when the solid is separated into several pieces. To present the concept, show a tall (12-oz.) container about half full of water and a piece of plasticine, any size.

WHAT WILL HAPPEN IF WE PUT THE PLASTICINE INTO THE WATER? (The level of the water will rise.)

After this has been checked by experiment, remove the plasticine and ask a child to mold it into a different shape.

WHAT WILL HAPPEN NOW WHEN IT IS PUT INTO THE WATER? (The water will rise.)

WILL IT RISE THE SAME AMOUNT AS BEFORE?

Let the children suggest an experiment to check the answer. Continue the activity until it is clear to the children that the amount of water displaced — the amount the water rises — does not depend on the shape of the piece of plasticine.

Activity B (Demonstration)

Place a tall (12-oz.) container on a saucer, and fill the container to the very top with water. Remove, with sponge or paper towels, any water that may have spilled into the saucer. Next, gently lower the plasticine you have shaped to fit a

Minneglass into the container, making sure that the overflow of water does not splash beyond the saucer, and that — with the plasticine in it — the water again reaches the top of the container. Remove the container from the saucer, being careful not to spill any water from it into the saucer. Empty the water in the saucer into a Minneglass. Now take the plasticine from the container and pack it into the Minneglass that was used to shape it. Have the children compare the two Minneglasses and lead them to the idea that the volume of water displaced is the same as the volume of the plasticine. Pour the Minneglass of water back into the container to show that it is completely refilled.

Activity C (Demonstration)

Partially fill a transparent container with water and mark the level of the water with a magic marker or a rubber band. Lower the shaped piece of plasticine into the water and mark the new level of the water.

WHAT HAPPENED?

WHY DID THE WATER RISE?

HOW MUCH DID IT RISE?

DOES THE AMOUNT THE WATER ROSE HAVE ANYTHING TO DO WITH THE VOLUME OF THE PLASTICINE?

To check the answers, do this: Remove the plasticine from the cylinder and fit it into a Minneglass. Fill another Minneglass with water and elicit the idea that the volumes of water and plasticine are the same. Pour the water from the Minneglass into the container. The children should see that the added water rises to about the same level as it did with the plasticine in it, and therefore the volumes are about the same.

Activity D (Demonstration)

Fill a tall container about half-way with water and mark the water level with a magic marker or rubber band. Show the children a dozen marbles, pebbles, washers, or other small objects.

HOW COULD WE FIND THE VOLUME OF THESE OBJECTS?

COULD WE USE THIS WATER SOMEHOW?

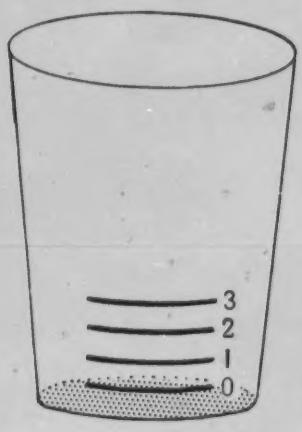
Guide the discussion so that the children see that the objects could be put into the water, the new level marked, the objects removed, and finally the numbers of Minneglasses of water needed to restore the new level determined. Let some children demonstrate how this is done.

CAN YOU THINK OF ANOTHER, EASIER WAY TO TELL HOW MUCH THE WATER RISES IN THE CONTAINER?

Try to elicit the idea that they could mark it like a measuring cup, or suggest the idea yourself.

Empty the container and mark the bottom of it 0, saying:

THIS IS THE LEVEL WHEN THERE ARE ZERO MINNEGLASSES OF WATER IN THE CONTAINER.



Have a child add a Minneglass of water and mark the new level with a line labeled 1.

THIS IS THE LEVEL WHEN THERE IS ONE MINNEGLASS OF WATER IN THE CONTAINER.

Have the children continue to add water, one Minneglass at a time, and label the levels until the container is full.

Empty out half of the water, mark, and record the level of the water in the container.

Volume of water > 3 Minneglasses

Volume of water < 4 Minneglasses

THE WATER LEVEL IS BETWEEN 3 AND 4 MINNEGLASSES.

Then gently add the marbles or other small objects, mark and record the new water level.

Volume of water > 6 Minneglasses

Volume of water < 7 Minneglasses

HOW MUCH DID THE WATER RISE?

If necessary, lead the children to count the number of volume units the water rose.

Some children may be able to get the number of volume units by subtraction:

In the case suggested above, either 6 - 3 or 7 - 4 shows that the change in the level was about 3 Minneglasses, though anything between 2 and 4 is possible.

A more complicated case would be:

First level — Volume of water $\doteq$ 5 Minneglasses

Second level — Volume of water > 8 Minneglasses
Volume of water < 9 Minneglasses

Change in level V — Volume of water > 3 Minneglasses
Volume of water < 4 Minneglasses

HOW MUCH DID THE WATER RISE? (Between 3 and 4 Minneglasses.)

WHAT IS THE VOLUME OF THE MARBLES? (Between 3 and 4 Minneglasses.)

Activity E

Distribute to each pair of children: 1 tray, 1 (12-oz.) cylindrical container, 1 container of water, 1 Minneglass, and a piece of plasticine (volume about 2 Minneglasses).

Have the children calibrate the cylinders by adding water, one Minneglassful at a time, and marking each level.

You may prefer to calibrate the containers yourself before distributing them. You can do this accurately enough by

calibrating one container, and then marking the others similarly. If calibrated cylinders are given to the children, lead them to check the calibrations.

HERE ARE SOME CYLINDERS WHICH I HAVE MARKED.

HOW CAN YOU CHECK TO SEE IF I HAVE MARKED THE LEVELS PROPERLY FOR MINNEGLASS UNITS OF VOLUME?

When the cylinders have been calibrated or checked, have each pair use theirs to determine the volume of the piece of plasticine. Give minimum guidance as to how this is to be done, referring to the preceding activity only when necessary.

Have the children try measuring the volume of other objects. Some objects that could be used for volume measurements are small attribute blocks, stones, small sponges, and plastic toys. If the children happen to pick an object that floats, let them try to work out their own solutions to the problem. Tell the children to retrieve an object by pouring the water back into their water container and catching the object. Then have them refill the cylinder to a different mark and repeat the measurement. Ask whether the starting level makes any difference in the resulting measurement of the volume of the plasticine.

Advanced students might like to experiment further. Have them make a hollow cylinder of plasticine (with a top and bottom) and poke a small hole through its top and bottom. When they put it in the water, they should discover that the water level rises quickly but then gradually changes to a new level. They should come to realize that the air must get out of the object before a good measurement of its volume can be obtained.

Activity F

As a review of the displacement of water, read "The Crow and the Pitcher" to the children and have them discuss it.

Your class may also enjoy trying to guess the following riddle, from Opie, Oxford Nursery Rhyme Book, Oxford Press.

Two brothers are we,
Great burdens we bear,
By which we are bitterly pressed;
The truth to say,
We are full all the day,
And empty when we go to rest.

THE CROW AND THE PITCHER

(Adapted from Aesop)

It was a very hot day. The sun shone brightly on a crow and made him very hot and thirsty. The crow flew high and low, looking for a brook or pond where he could get a drink of water. The longer he looked the thirstier he got, but he couldn't find a drop of water.

He looked and he looked and he looked!

At last he came to a small house in the middle of a huge field. On the ground by the side of the house there was a pitcher, and in the pitcher there was water!

The crow perched on the edge of the pitcher and poked his head down inside it to drink the water. But he couldn't reach down far enough. There was the water he had hunted for, but he still couldn't drink it! He was thirstier than ever.

He thought and thought about how to get at the water. Perhaps he could knock the pitcher over and catch at least a little of the water before it soaked into the ground. But he wasn't strong enough.

Then he tried to break the pitcher. He picked up a stone in his beak and flew up to drop the stone on the pitcher from above. The stone fell right down into the pitcher. The pitcher didn't break, but the water splashed up. The sight of it made him even more thirsty. He took another stone and that too fell into the pitcher. He saw that the water now was a little higher in the pitcher. Quickly he got more stones and dropped them one by one into the pitcher. The more stones he dropped into the pitcher, the higher the water rose.

At last the water was high enough. He perched on the edge of the pitcher and drank and drank and drank until he wasn't thirsty anymore.

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SECTION 4 MEASURING AND ORDERING TIME DURATIONS

PURPOSE

- To familiarize the children with language that indicates time relations and to have them recognize such references.
- To provide practice and develop skills in measuring time durations by counting, counting guided by a pendulum and by reading a clock.
- To introduce units for time duration (counts, swings, seconds).
- To introduce measures for time durations which can be used for communication in words and symbols.
- To develop an understanding of the conventional twelve-hour clock as a duration indicator.
- To give practice in telling time.

COMMENTARY

It has always been important to be able to measure time duration, but duration cannot be measured with a ruler or a container or a balance. The common way to measure duration involves counting or recording the repetitions of regular events. Perhaps man first reckoned a time interval by considering the period from sunrise to sunrise, from full moon to full moon, or from one spring flowering to another. The Indians of America counted years by "winters," months by "moons" and days by "sleeps." As man's ability to measure time durations improved, he became better able to recognize order and rhythms in the world around him. Measurement of duration is essential in order to carry on many activities in science as well as in everyday life.

In these lessons dealing with time duration, an event (or activity) is considered as a particular section of experience in which some specific action is performed or some other specified change occurs. As used in this unit, an event is considered to be any portion of our experience. Typical events might

include counting to ten, walking around the room, singing a song, reading a story, drawing a picture, the burning of a candle, the melting of an ice cube, or the swinging of a pendulum. An event has temporal extension somewhat as an object has spatial extension.

Duration is the property of an event which might be considered analogous to the property of length of an object. Thus, duration may be thought of as the quantity of time through which an event extends. Duration of events may be compared in somewhat the same way that lengths of objects are compared. The lengths of two objects may be compared by physically bringing the two objects together, side by side, with a pair of ends aligned. That object which extends beyond the other has the greater length. In an analogous manner, a direct comparison of durations of concurrent events may be accomplished by having both events start simultaneously. That event which continues after the other has ceased has the greater duration.

If two objects cannot be brought together for direct comparison of lengths, they may be compared indirectly by using a set of objects with standard lengths, such as paper-clip chains or rulers, which can be directly compared with each of the objects to be measured. (It is a matter of definition that the lengths of the standard objects remain constant when they are moved.) Similarly, if two events cannot be made concurrent in order to compare their durations directly, the durations may be compared indirectly by using a set of events of standard duration, such as the number of ticks produced by a clock while each event is going on. In this section, the set of events of standard duration is the number of counts (from zero to various numbers), the counting being guided by periodic events of a physical system such as the swinging of a pendulum. (Again it is a matter of definition that the successive swings of a pendulum have equal durations.) Thus, duration may be measured by counting the repetitions of uniformly repetitive events.

A clock is defined as any physical periodic system used to guide counting. The idea of a clock is much more general than the twelve-hour clock with which we are most familiar. An attempt is made here to present this more general notion of measuring duration without presenting the familiar clocks and calendars as the only measuring guides.

The similarity of duration to length is an important idea in understanding the new concepts of space and time. Building on the concepts of measurement introduced earlier, these lessons establish a foundation for the modern ideas of time.

Another analogy between duration and length is noted here. Two events are said to have the same duration if comparison does not reveal a difference in duration. This depends upon the sensitivity of the comparison to detect the difference (inequality) of durations just as the designation "length is the same" depends on the sensitivity of the measurement of length.

The before-after relationship between events may be defined in terms of memory. If one event is being perceived directly and another event is remembered, we say that the remembered event preceded, or occurred before, the directly perceived event. To a single observer, events can be arranged in a linear sequence. If A was observed before B, and B was observed before C, we are always sure that A was observed before C and that the observations were in the order A, B, C. Such a sequence is said to be "time-ordered." Thus time-order is a transitive relation; on the basis of the time-order of A and B and the same order of B and C, we can determine the order of A and C.

Clocks can be designed to count automatically or their readings can be translated into a time-ordered sequence to which numbers can be assigned. Other events occurring simultaneously with the clock events (number production) can be labeled with the numerals of the clock events. These events may be time-ordered by simply arranging them according to the magnitude of the assigned clock numbers. Thus an event associated with number three (three o'clock) occurred before an event associated with number five (in the same clock sequence). Clocks, then, fulfill two time-related roles. They may be used to measure duration by counting the number of repetitions (clock events) that occur during an event, and they may be used to place events in a time-ordered sequence.

The activities of this section proceed in much the same fashion for duration comparisons of events as did the activities for length comparisons of objects in Section I. To begin with, the children are led to see that the duration of the event of counting to five, represented by "C5", is less than the duration of the

event of counting to ten, represented by "C10." This relationship may be expressed as "C5 < C10." The following series of standard durations can be constructed:

$$C1 < C2 < C3 < C4 < C5 < C6 < C7 < C8 < C9 < C10$$

This sequence assumes that the same steady counting rate is used in counting to ten as was used in counting to five, etc. The sequence should be read, "The duration of counting to one is less than the duration of counting to two, which is less than the duration of counting to three," etc.

When counting is used to measure the duration of an event such as drinking a glass of water, and the event which started at zero count ends between two numbers such as twelve and thirteen, we express this result: C12 < duration of drinking a glass of water < C13. This is read: "The duration of counting to twelve is less than the duration of drinking a glass of water, which is less than the duration of counting to 13," or "Drinking a glass of water takes a greater duration than counting to twelve and a lesser duration than counting to thirteen." If the end of an event and a count seem to occur together, such as when the duration of "drawing a triangle" ends on the count of nine, we express this result: Duration of drawing a triangle $\doteq$ C9. This is read: "The duration of drawing a triangle appears to be (is approximately) equal to the duration of counting to nine." We might also express this as: "Drawing a triangle takes about the same duration as counting to nine."

In order to standardize the rate of counting, the pendulum is introduced and used as a guide for counting (i.e., as a clock). The durations of several events are measured, with the pendulum regulating the counting rate.

THE PENDULUM

Pendulums for classroom use are easy to make. The bobs or weights for the pendulums can be nuts, washers, fishline sinkers, or drapery weights. Although experiments have shown that the period of the pendulum (the duration of one swing) does not depend upon the amount of material in the bob, all pendulums used in this part should have bobs of the same type. The bobs for the various pendulums used in Lesson 24 should be

painted with different colors or marked with colored bits of paper so they can be identified by name.

The pendulum bob should be suspended by a length of strong thread. Tie one end of the thread to the bob and cut off the excess thread. The other end of the thread may be tied to a paper clip. The length of the pendulum is measured from the point of suspension (the paper clip) to the center of the bob. The paper clip can be fastened to the end of the ruler with a thumb tack.

To permit the bob to swing freely without hitting anything, the supporting ruler may be extended from a desk or shelf, with the other end held down by tape or books. A short pendulum can be taped to a chair seat, as shown in Figure 1. The rod should not extend so far from its support that it vibrates when the pendulum swings. An extension of 3 inches is suggested.

Scientists define the period of a pendulum as the time required for the pendulum to make a complete swing from its starting point to the other side and back again. This is described in Lesson 22 (page 144). A 10-inch pendulum gives a 1-second period.

Pendulum Construction

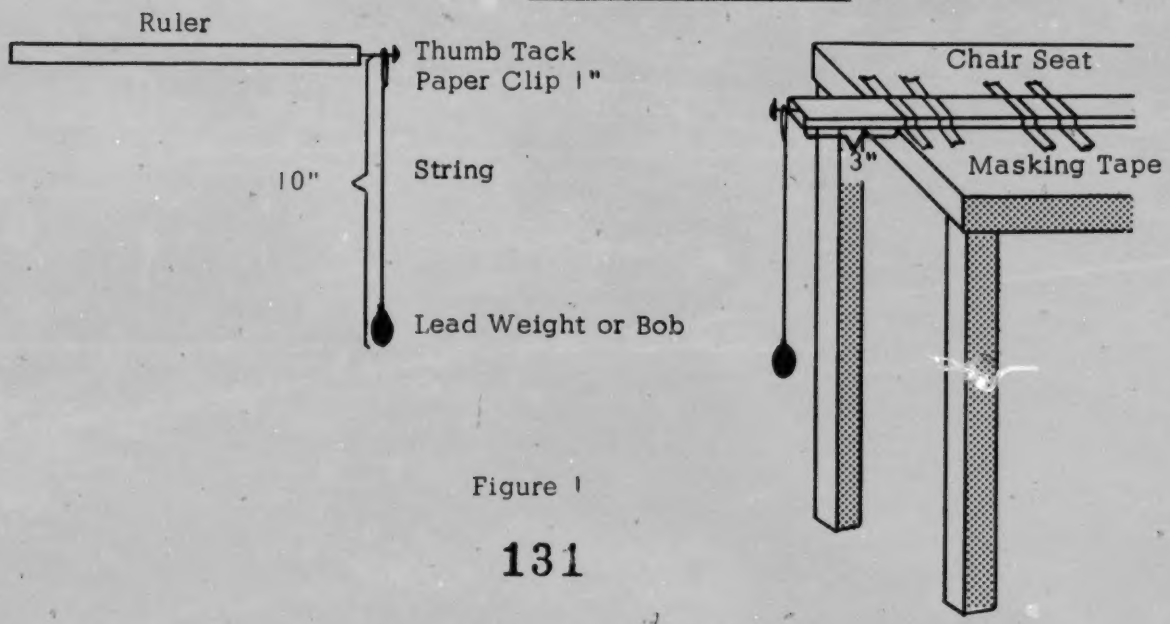
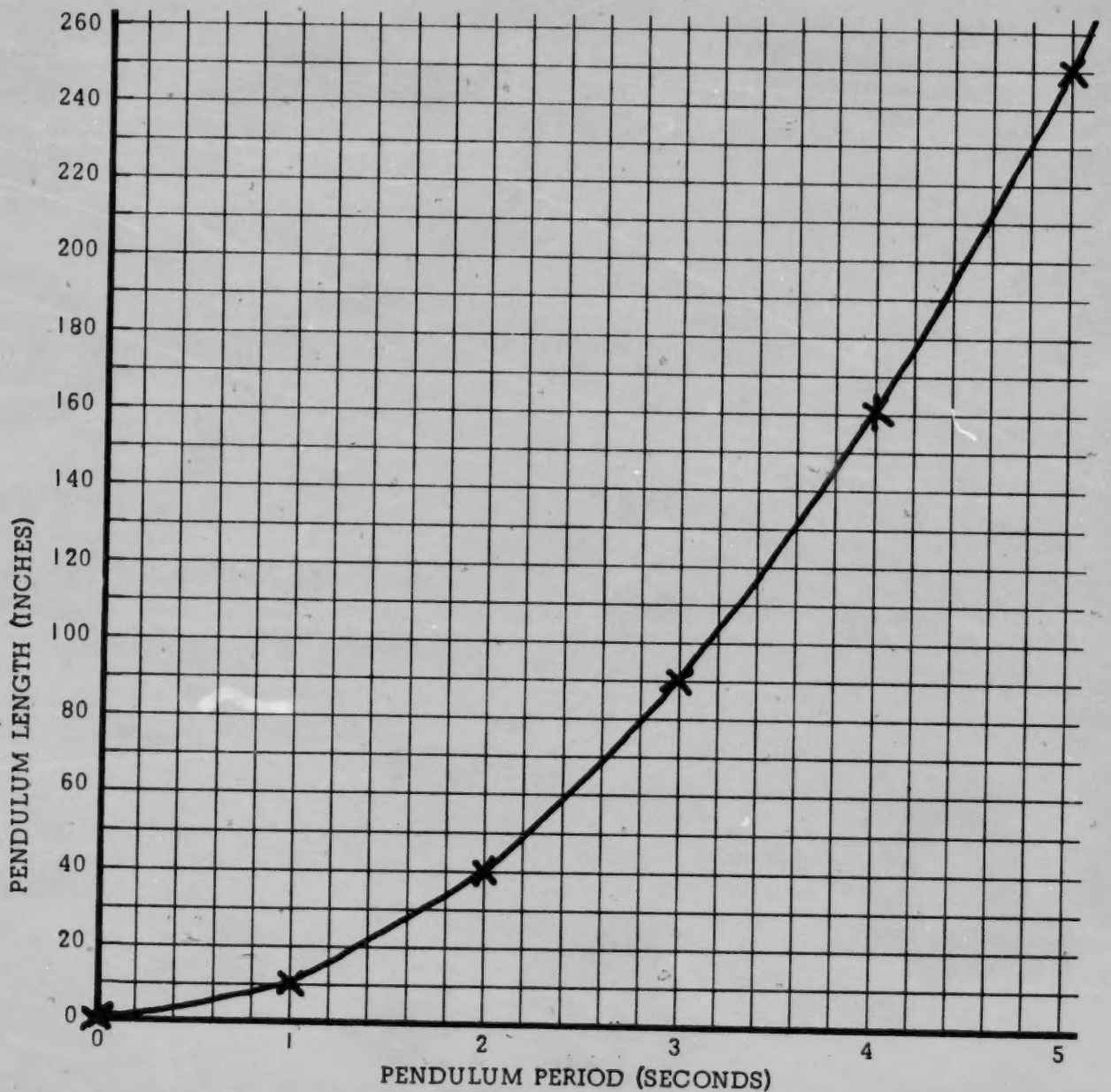


Figure 1

Pendulums of greater length, and consequently greater period, can be made by attaching the thread to fixtures or pipes on the ceiling of the classroom. Figure 2, below, shows the relationship between the period (in seconds) and the length (in inches) of a pendulum. The curve shows that a pendulum 40 inches long will have a period of approximately two seconds.



Detailed directions for using the pendulum are given in Lesson 25.

Figure 2

Lesson 24 involves an experiment in which the children receive more experience with pendulums while they are discovering the relation between the length of the pendulum and the duration of its period. They are also introduced to the idea of making predictions on the basis of data which they have collected.

A poem in Lesson 25 suggests that many devices can be used to regulate counting — they can be used, in fact, as clocks. Several such "strange" clocks are discussed before the conventional twelve-hour clock is introduced in Lesson 26. The minute is defined as the duration of one revolution of the second hand and the minute hand is shown to be a recording device for revolutions of the second hand. The hour and the hour hand are similarly related to the minute and minute hand, and the children are given experience in using the clock for measuring durations in Lesson 27. Finally, in Lesson 28, telling time is associated with the durations since noon or midnight as shown on the clock.

Lesson 17: COMPARING THE DURATION OF ACTIVITIES

This lesson introduces the concept of time duration and the ways of comparing time durations qualitatively. The children should learn to compare and communicate differences in the durations of several pairs of activities.

The story, "Which Cookies Today?" contains many references to time duration and time order. You will be using it to determine whether the children can identify the references to durations, before-after relationships, and concurrent activities. The exercise should demonstrate to the children that concepts of time and change are involved in almost every aspect of their lives.

In order to check the children's comprehension of the concept of time duration, several pairs (or sets of three) events are listed in Activity B. These are useful in determining whether the children can properly order the events according to their durations. For example: In the first pair of events, one child writes a word while another child says the same word. (Both start at the same time.) By using it consistently yourself, encourage the children to use specific language, e.g., "The duration of writing the word is greater than the duration of saying the word." However, allow the children to say "The time for writing ..." etc. Throughout these activities, you will have to make it clear — again and again — that these are not races. You are all just trying to compare the durations of some events.

MATERIALS

- story, "Which Cookies Today?"

PROCEDURE

Activity A

Before reading the story, "Which Cookies Today?", explain that it often mentions ideas the children are going to study for the next few days — time duration (how long an event took) — and time order (which event came first).

WHICH COOKIES TODAY?

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Story by Elaine Vogt
Illustrations by Sonia Forseth

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WHICH COOKIES TODAY?

Many years ago there was a very spry old grandmother who liked to bake cookies and take them to all her grandchildren. She would mix, mix, mix the batter. Then she would roll, roll, roll the dough. Then she would bake, bake, bake the cookies.

And then she would take off her apron, tie on her bonnet, hop on her bicycle and deliver the cookies. It was a bicycle such as you have never seen! It had a large wheel in front and a small wheel in back. She would pedal, pedal, pedal down the long winding road and stop at each of the houses where her grandchildren lived. And the children were always waiting for her while she pedalled down the road, because she came at the same time every day.

Five days a week Grandma baked cookies.

On Mondays she always baked chocolate cookies.

On Tuesdays she always baked peanut butter cookies.

On Wednesdays she always made oatmeal cookies.

On Thursdays she always made date cookies.

On Fridays she always made sugar cookies.

On Saturdays and Sundays she didn't make any cookies at all. She would oil and polish her bicycle, and then she would just sit in her rocking chair and rest until Monday morning. Then she would start bright and early on the chocolate cookies.

Her cuckoo clock would wake her when the sun had just come up over the horizon. Each day it took Grandma two hours to bake her cookies and pack them in the six little baskets she kept just for that

purpose. Then off she went to deliver the cookies. First she went pedal, pedal, pedal to Julie's house. Julie was only three years old and took a nap in the morning, and Grandma liked to see her every day before she went to sleep. (Of course Julie had the littlest cookie basket.)

Then she went pedal, pedal, pedal to John's house. (His cookie basket was bigger because he had a much larger appetite.) John was four years old, and he was always waiting for Grandma with his nose pressed against the window.

Then Grandma hopped on her bicycle again and rode like the wind to Susan's house. Susan lived quite a bit further down the winding road. If it were Wednesday, Susan was sure to be waiting by the side of the road. She especially loved oatmeal cookies. (Other days she waited inside the house.)

Then Grandma went on, pedal, pedal, pedal down the road to the house where Peggy and Barbara lived. On Mondays Peggy would be sitting waiting on the front steps, but Barbara would be inside saying, "I don't like chocolate cookies." It was quite a different story on Thursdays, though, for then Barbara would be on the front steps waiting eagerly, and Peggy would be inside grumbling that she didn't like date cookies.

Then Grandma went pedal, pedal, pedal down the road to Jimmy's house. She stopped for a little lunch there, because by then it was getting to be noon. Jimmy came home from school and they had lunch together. They always talked while they ate lunch. Jimmy liked all the different kinds of cookies that Grandma baked. He thought Grandma was the greatest cookie-baker in the whole world,

and he said so every day. Grandma liked to hear him say this. She said it perked her up for her long bicycle ride home.

One morning Grandma woke up and looked around her, very puzzled. The sun was high in the sky, not just coming up as it usually was when the cuckoo clock woke her in the morning. She was even more puzzled when she realized that the cuckoo clock hadn't awakened her at all. There was the poor bird hanging lopsided off his perch. It had just cuckooed itself out at midnight the night before. Cuckoo, cuckoo, twelve times, and then the spring had broken and it couldn't cuckoo any more!

Grandma had no idea what time it was, but she knew it was terribly late. At noon the sun would be right overhead, and look how high it was already! So she went flying about the kitchen trying to get everything done at once. She was so confused that she couldn't even remember what day it was. What kind of cookies was she supposed to bake? Did she have time to bake any cookies at all? But her grandchildren would be so disappointed. So she decided to bake anyway — but which cookies? It must be Thursday, because yesterday most certainly was Wednesday, wasn't it? She wasn't at all sure about the day, but she seemed to remember that she had baked oatmeal cookies yesterday. Hadn't Susan been waiting for her by the side of the road when Grandma came pedalling along?

So Grandma got busy making date cookies. She didn't even stop to dress. Still in her night cap and long ruffled nightgown she flew about the kitchen, chop, chop, chopping dates, beat, beat, beating eggs, mix, mix, mixing dough. When she was finally ready to pop the cookies into the oven the sun was directly overhead.

"Noon," she thought. "My poor little Barbara, she loves

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my date cookies so much, but they will never be there in time for her lunch!" She ran into the bedroom to get dressed while the cookies baked, and then rushed off on her bicycle, leaving the kitchen in the worst mess you ever saw.

She went puff, puff, puffing down the road, pedaling away as fast as she could. At last she came to Julie's house. There in the road stood Julie's mother waiting and holding little Julie by the hand. Julie had already had her nap and was up again.

"Wherever were you? We were so worried!" said Julie's mother.

"Huff, puff," said Grandma. Then she told how the cuckoo clock had cuckooed its last cuckoo, and how she never got up 'til the sun was high in the sky. Then quickly she left the cookie basket and picked up the empty one from the day before, and on she went down the road.

Puff, puff, puff, she pedalled away to John's house. And sure enough, when she came round the bend, there were John and his mother standing and watching for her.

"Wherever were you? We were so worried!"

Grandma huffed and puffed and told them how the cuckoo clock had cuckooed its last cuckoo, and then she gave them their cookies and rode on down the winding road to Susan's house. Of course you can guess who was standing out watching for her.

"Wherever were you? We were so worried!"

So she huffed and puffed and told them how the cuckoo clock had cuckooed its last cuckoo, and then gave them their cookies and rode on down the winding road.

Thursday was the day for Barbara to be waiting on the front steps while Peggy sat indoors grumbling that she didn't like date cookies. But you can be sure that they were both standing watching for Grandma, and their mother was there too.

"Wherever were you? We were so worried!"

Huff, puff, and all about the cuckoo. And then on to the last stop.

Of course Jimmy wasn't waiting for her, because he had to go back to school after lunch. But his mother was there out on the road. She helped Grandma off the bicycle and into the house. (Grandma was awfully tired by now, with all the rushing around she had been doing.) No questions until a big bowl of soup had been eaten down to the last drop. And then one last time she told the story of how the cuckoo had cuckooed its last cuckoo.

Then she said, "Well, now I feel rested, and I'll go on home again and clean up the mess I left in my kitchen."

So that's what she did. And afterwards she sat in her rocking chair on the front porch and had a nice little snooze. When she woke up the sun was just going down, and she thought it must be time for her to make herself some supper. While she was thinking what to make, she heard a noise down the road.

There coming around the bend was a little parade. All her children and all her grandchildren were there. First came Jimmy, carrying a plate with a piece of grapefruit on it, and a big red cherry sitting right in the middle.

Then Barbara came, carrying a dinner plate. It was covered,

so Grandma couldn't guess what was on it, but steam was coming out around the edges, and she just knew it would be something good to eat. Peggy was carrying a salad bowl, and John had a great big chocolate cake on a platter — a chocolate cake big enough to feed a whole parade!

And last of all was little Julie carrying a big box. They all crowded around Grandma and kissed her. Grandma surely had forgotten what day it was. Not only was it Thursday, but it was her birthday. It had been her birthday all day long!

Can you imagine forgetting your own birthday? They made her open her present, and guess what it was — a bright and shiny clock, with two big bells on the top, bells that would ring to wake her at the same time every morning.

And do you know, she was never late with her cookies again!

Activity B

To start the children at comparing the time durations required for various events, select several of the suggestions below or make up others that may be more appropriate for your class. In some cases, two or three children will be carrying out the activities, starting at the same time, while others watch to determine which event took longer (which event was going on after the other had stopped.)

Possible events for comparison:

Writing a word ----------Saying the word

Drinking a glass of water ---------- Walking around the room

Tying a shoestring ---------- Buttoning a button

Drawing a square ---------- Drawing a house

Walking one block ---------- Walking three blocks

Choose a word and two children for this event. Both should start at the same time.

WATCH CAREFULLY TO SEE WHICH EVENT — WRITING A WORD OR SAYING THE WORD — HAD THE LONGER DURATION.

After the word has been said and written, have the class discuss the relative durations of the two events.

WHICH HAD A LONGER DURATION — WHICH WAS STILL GOING ON — AFTER THE OTHER HAD STOPPED? (Writing the word.)

YES, THE DURATION OF WRITING A WORD WAS GREATER THAN THE DURATION OF SAYING THE WORD.

For events which can not be carried on at the same time, have the children estimate which had a longer duration.

WHICH WOULD HAVE A LONGER DURATION — WALKING AROUND ONE BLOCK OR AROUND THREE BLOCKS?

When three events are considered together, have the children attempt to list them in the order of the length of their durations with the shortest duration first.

Events whose durations might be ordered:

Brushing your teeth — Eating your breakfast — Walking to school

Reading a story — Reading a poem — Recess

Noon lunch period — Recess — Music Period

Germination of a seed — Evaporation of water in a pan — Duration of a school day

WHICH HAD THE GREATEST DURATION — BRUSHING YOUR TEETH, EATING YOUR BREAKFAST, OR WALKING TO SCHOOL?

COULD WE PUT THE THREE DURATIONS IN ORDER?

Summarize the children's answers.

THE DURATION OF BRUSHING YOUR TEETH WAS LESS THAN THE DURATION OF WALKING TO SCHOOL WHICH WAS LESS THAN THE DURATION OF EATING BREAKFAST.

Lesson 18: ORDERING TIME DURATIONS

Counting as a method of measuring durations is developed in this lesson and provides a background for the concept of clocks in later lessons.

To introduce the work on recording the ordering of durations you will need a series of "duration" cards. You can make these from 3" x 5" or 4" x 6" index cards or other cardboard. They should have printing large enough to be legible from the chalktray. Cards should be marked C1, C2, etc. You should make as many cards as there are children in your class.

MATERIALS

- set of duration cards
- order cards (<) from the number card set, one per child
  (Reverse to have > symbol)

PROCEDURE

Activity A

WHICH TAKES A GREATER DURATION — COUNTING TO FIVE OR COUNTING TO TEN?

Some children will probably suggest that counting to ten has the greater duration.

CAN YOU SHOW US THAT IS SO?

Give one child the duration card, C10.

THIS CARD REPRESENTS THE DURATION OF COUNTING FROM ZERO (OR START) TO TEN.

Give another child the C5 card.

WHAT DOES THIS CARD REPRESENT? (The duration of counting from zero to five.)

Have the two children stand in the front of the room and count in unison from zero to the number indicated by their cards.

COUNT TOGETHER FROM ZERO UNTIL YOU REACH YOUR NUMBER. THEN STOP COUNTING AND KNEEL.

The children should not race, but say each count loudly and clearly. Counting in unison may have to be practiced. Indicate a signal for starting. Keeping the beat for the children, as in music lessons, is helpful. When the children finish the count, ask the rest of the class what they observed.

WHICH EVENT HAD THE LONGER DURATION — COUNTING TO FIVE OR COUNTING TO TEN? (Counting to ten.)

HOW DO WE KNOW? (Both started counting together and they reached five first.)

Summarize the result by having the children put the two cards on the chalktray and then have a third child place an order card in the right direction between the other cards.

C5	<	C10

WHAT DOES THIS TELL US? (The duration for counting to five is less than the duration for counting to ten.)

CAN SOMEONE SHOW THIS ANOTHER WAY?

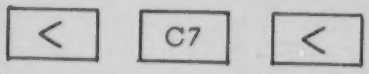
C10	>	C5

WHAT DOES THIS TELL US? (The duration for counting to ten is greater than the duration for counting to five.)

Show the class the C7 card and have the children count in unison. Have pairs of children choose other cards and carry out the first activity, ordering and displaying the cards properly, and reading back the relations, e.g.:

Activity B

Remove all the cards from the chalktray. Place the C7 card on the tray with the "less than" symbols on both sides. Give a child the C6 card.



CAN YOU PLACE THIS WHERE IT SHOULD BE ON THE TRAY?

Have another child place the C8 card.

WHAT DO WE NEED NEXT ON THE TRAY? (A "less than" symbol.)

WHAT COUNT CARD COULD WE USE? (C5 or C9.)

WHO CAN FIND THESE CARDS AND PLACE THEM PROPERLY?

When two children have done this correctly, cards show:

NOW WHO CAN PLACE THE CARD FOR THE DURATION FOR COUNTING TO THREE, "C3"?

When this is done, continue with the cards in this order:

C11, C2, C10, C17, C20, C14, C1, C4, C15

The children will find that they will have to move some cards over to include the new duration and order cards.

The sequence should now be:

C1 < C2 < C3 < C4 < C5 < C6 < C7 < C8 <

C9 < C10 < C11 < C14 < C15 < C17 < C20

Note that only 15 cards are used. Some are missing. This is done so that the children will see that the ordering does not require a complete series of numerals.

At this point in subsequent work you may wish to have the children reverse the order of the duration cards and practice reading the record with the "is greater than" relation.

C20 > C17 > C15 > C11 > C10 .....

"The duration of counting to 20 is greater than the duration of counting to 17 ....."

Activity C

Once the sequence in Activity B has been constructed, ask the children to extract binary relations from it.

FROM OUR RECORD, WHAT DO WE SEE ABOUT THE DURATION OF COUNTING TO SIX AND THE DURATION OF COUNTING TO NINE? (The duration of counting to six is less than the duration of counting to nine.)

WHO CAN WRITE IT ON THE CHALKBOARD? (C6 < C9.)

Have the children carry out several of these deduced binary comparisons.

It is important for the children to realize that they can thus apply the transitivity principle to extract these binary relations from such a time-ordered sequence. They should be able to construct the ordered sequence of cards by the value of the numerals and read aloud, in sentences, the expressions as relationships between the durations of the counting activities, e.g., "The time duration of counting to six is less than the time duration of counting to nine," and "The time duration of counting to nine is greater than the time duration of counting to six."

If it seems appropriate at this time, have the children review time ordering.

DID C9 END BEFORE OR AFTER C6?

DID ANY TWO OF THESE EVENTS END AT THE SAME TIME?

Lesson 19: COUNTING AS A MEASURE OF TIME DURATION

This lesson provides experience in the use of rote counting for measuring time duration, and practice with the notation for recording the measurements.

You will ask the children to arrange themselves in a line according to the duration of the counting done by each one. This should strengthen their understanding of how counting can be used to measure durations.

MATERIALS

- duration cards, one for each child, from Lesson 18
- order cards, one for each child

PROCEDURE

Distribute a duration card to each child in a random manner but from a consecutive set (C1...), and have the set of order cards available. Have each child stand by his desk with his card in his hand. Have the class count in unison, "zero," "one," "two," etc. After each count, the child holding the card representing that count should sit down and stop counting.

When all are seated:

> WHO COUNTED FOR THE LEAST DURATION? (The child with the C1 card.)

Have this child come forward with his card and stand at one side of the room.

> OF THE CHILDREN SEATED, WHO COUNTED FOR THE NEXT GREATER DURATION? (C2.)

Have this child, holding his card, stand behind the first child. Continue this process until all the children are standing in the line.

Pick any two children in the line.

> DID MARY COUNT FOR A GREATER OR A LESSER DURATION THAN JOE?

Do this for several pairs. Then review the recording procedure.

WHICH OF YOU COUNTED FOR THE GREATEST DURATION OF ALL? (The child at the end of the line.)

Have this child place his card in the chalktray and sit down.

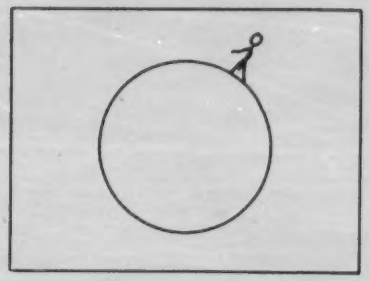
OF THOSE CHILDREN STANDING, WHO COUNTED FOR THE NEXT LESSER DURATION? (The child next to the end of the line.)

Have this child place his card and a "less than" card appropriately in the chalktray and take his seat. Repeat this until all the children are seated. The discussion of this record should emphasize its relation to the actual counting process which it symbolizes and the durations of the various counting activities of the various children.

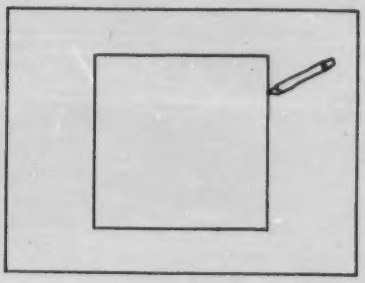
Lesson 20: COUNTING TO COMPARE TIME DURATIONS

This lesson has the children use counting to compare the durations of two activities which cannot be started together. The situation in which direct comparison is impossible is created by asking the children to compare the durations of two activities carried out by one child, activities which he cannot carry on simultaneously. The counting technique with duration cards used in Lesson 19 is continued with the addition of sheets of paper on which there are symbols representing the activities whose durations are to be compared.

For example walking around the room can be depicted as:



Drawing a square can be shown as:



These sheets can be included at the appropriate places in the ordered duration cards and the relative durations determined by applying transitivity.

MATERIALS

- duration and symbol cards from Lesson 19
- 2 sheets of paper (9"x 12") with an activity depicted on each, as shown above. Use a felt marker for the drawing.
- other materials for Optional Activities (See p. 141.)

PROCEDURE

Give one child the sheets representing the activity of drawing a square on the chalkboard and representing the activity of walking around the room. Ask the child what each of the cards represents. Then ask if it would take more time for him to draw a large square or for him to walk around the room. Ask him to check his answer by a direct comparison of the durations of the two activities. He will only be able to guess which takes a longer duration.

Ask the children for suggestions of ways to find which duration is greater. Hopefully, someone will suggest counting during each activity separately and then comparing the counts. Whether or not this suggestion is made, carry out the next exercise.

Give each child at his seat a duration card, each with a different numeral. Recall to them how they counted with the cards in Lesson 19. Have all the children stand with their cards. Instruct the child at the front of the room to take the "circle" card with him and to be prepared to do the activity represented by the card. Now locate a line on the floor as a starting and finishing line. Tell the children they will count in unison, each sitting down after he has counted the duration indicated by his card. Tell the class that the child with the circle card will be walking around the room while the rest are counting, and to observe carefully whether the activity of counting took more or less time than it took the child to walk around the room.

WATCH SUE (who will be the walker) CAREFULLY AND NOTICE WHETHER SHE IS STILL WALKING WHEN YOU SIT DOWN.

When the child starts the walk, all the children say "zero" and continue the counting in unison and the sitting down (as in Lesson 19) until all are seated. If the count is finished before the child stops walking, repeat the activity, increasing the walking rate and slowing down the counting.

WHICH OF YOU WERE STILL COUNTING AFTER SUE FINISHED WALKING?

WHO COUNTED FOR A LONGER DURATION THAN THE DURATION OF HER WALK?

Have all who counted for the longer duration stand on one side of the room.

WHICH OF YOU FINISHED COUNTING BEFORE SUE FINISHED WALKING?

WHO COUNTED FOR A SHORTER DURATION THAN THE DURATION OF HER WALK?

Have all who counted for the shorter duration stand on the other side of the room. If a child sat down just as Sue finished (his counting duration was about the same as the walking duration), have him stand between the two groups with the walker.

To the group with the larger numbers:

DID YOU ALL COUNT FOR A GREATER DURATION THAN IT TOOK SUE TO WALK AROUND THE ROOM? (Yes.)

WHICH OF YOU COUNTED FOR THE SHORTEST DURATION? (The one with the smallest numeral on his card.)

Have that child take his card and stand at the left side of the walker.

To the group with the smaller numbers:

DID ALL OF YOU COUNT FOR A LESSER DURATION THAN IT TOOK SUE TO WALK AROUND THE ROOM? (Yes.)

WHICH OF YOU COUNTED FOR THE LONGEST DURATION? (The one with the largest numeral on his card.)

Have that child take his card and stand at the right side of the walker. Ask the three children to place their cards appropriately in the chalktray. The situation might be:

C18 < ♂ < C19

The other children may wish to place their cards in the chalk-tray, separated by order cards so that all the cards are in the order of their durations.

C1 < C2 <· · · · C17 < C18 < < C19 <· · · · C23 <· · · ·

Supply each child with a blank card and have him make a duration card like the one he placed in the tray. Give the card representing drawing a square to Sue (the walker), have her come to the chalkboard and draw a square there, and repeat the former procedure for the measurement of the duration of this activity. The grouping and placing of the cards should be done as before, but the duration cards should now be placed in front of their mates already in the tray. Room will have to be made for the card representing the new activity and a few order cards added. If the new activity results in:

C6 < < C7

the final series would be

C1 <· · · · C6 < < C7· · · · < C18 < < C19 <· · · · C23 <· · · ·

from which we conclude (by transitivity) that:

<

"Drawing the square has less duration than walking around the room."

Review the lesson with the class.

WHAT WERE WE REALLY TRYING TO FIND OUT WHEN WE HAD SUE WALK AROUND THE ROOM AND DRAW THE SQUARE? (We were trying to find which took longer to do -- which had the longer time duration.)

HOW DID WE MEASURE THE TIME DURATIONS OF THESE EVENTS? (By counting.)

WHAT DID WE USE TO HELP US REMEMBER THE TIME DURATIONS? (We used cards.)

WHAT DID WE DO WITH THE CARDS? (We put them in order in the chalktray, with that for the least duration at the left and that for greater durations to the right.)

WHAT DID WE PUT BETWEEN THE DURATION CARDS? (We put "less than" cards between them.)

IF AN EVENT CARD IS TO THE LEFT OF ANOTHER ONE, IN THIS KIND OF ORDERING, WHAT DO WE KNOW ABOUT IT? (The event it tells about took a shorter time -- had a lesser duration -- than the ones to the right of it.)

Point to the card at the extreme right:

DOES THAT MEAN THAT ALL THE CARDS TO THE LEFT OF THIS CARD SHOW EVENTS THAT HAD LESSER DURATIONS THAN THE EVENT THAT THIS CARD STANDS FOR? (Yes, the farther to the left we go, the less time duration is shown, because we are using "less than" symbols.)

Point to the card at the extreme left:

DOES THIS CARD SHOW A SHORTER -- LESSER -- TIME DURATION THAN ALL THE OTHERS? (Yes.)

WHY? (Because that is the way we arranged -- ordered -- the cards.)

COULD WE MAKE OUR ORDER SAY THE SAME THING -- THAT THIS DURATION IS LONGER THAN THAT ONE -- IN ANOTHER WAY? (Yes, we could use the "greater than" signs and turn the whole order around.)

Have the class do this. Then have them order duplicates of any two cards you select from the entire row.

OPTIONAL ACTIVITIES

This page provides suggestions for more practice with regular counting to compare time durations of non-concurrent activities. (Some of these activities may be omitted if you feel them unnecessary on the basis of preceding work.)

This is essentially a repeat of Lesson 20 except that the children remain seated, and duration cards are not used. The children count in unison and remember the numbers which were said just before and just after the activities are finished. The results are recorded in a notation similar to that of Lesson 18. The durations of two or more activities are compared.

Any set of activities from the following suggestions, or ones which occur to you or the children, are to be compared and recorded as in Lesson 19. When the activities involve a child, they should be done successively by the same child so that the possibility of direct comparison of the durations is removed. Comparisons could be made of the durations needed:

1. For a child to return blocks (Minnebars, Tinkertoys, etc.) to a box and to do some other pick-up activity.

2. For a child to pass out napkins to the class and to do some other distribution or pick-up activity.

3. For a child to empty water from a quart bottle (or any with a small neck) with the bottle held on its side and held upside down.

4. For a given amount of water to disappear when watering each of two plants.

5. To play each of two short records on the record player.

6. To sing each of two songs.

Lesson 21: STANDARD UNITS OF TIME DURATION

This lesson develops the need for a standard unit to measure time duration.

By having the children count silently in comparing durations or asking them to compare the durations of activities done on different days, etc., they are led to see the advantage of having their counting guided by some "clock" — any system which provides a sequence of events of dependably regular duration which can be used as a guide for counting.

MATERIALS

- wind-up toys, music boxes, etc.
- Worksheet 20

PROCEDURE

Have the children measure the duration for an event such as the running down of a wind-up toy by counting together and recording the results on Worksheet 20. If the toy stopped between the counts of, say 15 and 16, they should complete the first problem on the worksheet as follows:

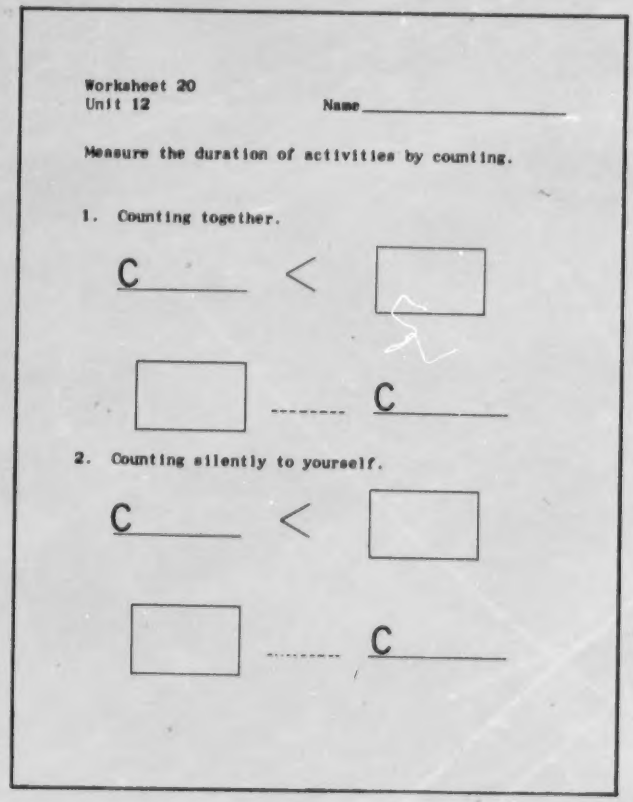
C15 >

< C16

If the toy stopped on the count of 23, they should complete the first line thus:

C23 ≐

This may be read: "The duration for the car to run down is about the same as the duration of counting to 23."


Worksheet 20
Unit 12 Name _____

Measure the duration of activities by counting.

1. Counting together.

C _____ < ☐

☐ ------- C _____

2. Counting silently to yourself.

C _____ < ☐

☐ ------- C _____

For the next activities (which may be selected from Lessons 17 or 21), each child should count silently to himself, then record his result.

THIS TIME WHILE THE TOY IS RUNNING EACH OF YOU SHOULD COUNT TO YOURSELF, SILENTLY, AND WRITE DOWN YOUR LAST COUNT ON YOUR WORK-SHEET.

When they have made their records, ask various children for their results and list them on the board. It is probable that there will be differences in the results. Ordinarily the children will count at different rates, but because of the practice in unison counting and the fact that they may not be able to count very quietly, they may all have arrived at the same number. If this happens, choose two children who normally would have different counting rates and let them count silently and independently, facing away from each other, while the activity is performed.

When different counts are recorded, guide the discussion to bring out the fact that different people may count at different rates, making comparisons questionable.

WHY DID WE GET DIFFERENT RESULTS? CAN ANYONE GUESS? (Perhaps we counted at different rates.)

The need for a guide for uniform counting can be made even clearer by having the children use counting to measure the duration of the same activity at two different times. Then raise the question, "Are we sure that we counted the same way -- just as fast or just as slowly -- each time?"

Such activities will probably be sufficient to show the need for a reliable guide for counting. Ask for suggestions as to some such guide. Then show a pendulum swinging.

WE WILL USE THIS TO HELP US COUNT IN OUR NEXT LESSONS.

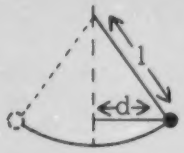
FOR TOMORROW, SEE IF YOU CAN THINK OF A WAY THAT WE COULD USE THIS PENDULUM TO HELP US ALL COUNT THE SAME WAY EACH TIME WE MEASURE A DURATION.

Lesson 22: THE PENDULUM AS A COUNTING GUIDE

The activities in this lesson acquaint the children with the pendulum as a guide for counting and give them practice in counting with its swings.

Instructions for making the pendulums needed in the following lessons are given in the section commentary, pp. III-3. For the first use of the pendulum, it is preferable to use one that is five or six feet long so that the duration of a complete swing (period) is more than two seconds. The string can be attached to a light fixture or a support rod can be attached to the top of a door, etc.

To start the swing of the pendulum, hold the bob so that the arm (string) is less than 30 degrees from the vertical. To keep the angle of the pendulum cord with the vertical less than 30 degrees, keep the distance (d) from the release to the vertical less than half the length (l) of the pendulum.



Release the bob gently without pushing. If the arc of the swing is too great, the duration of the swings of the pendulum will change significantly as the length of the arc decreases. When counting the swings, the counter should say "Zero" as the pendulum is released, and then "One" when it first returns to its release position, etc. (one count for each complete swing -- over and back).

Be sure to experiment with a pendulum yourself before showing it to the class. As you demonstrate its use and as the children use it to guide their counting, emphasize that:

1. The arc of the swing should be kept small. (Demonstrate this several times.)

2. The bob must be released gently -- without pushing -- simply by opening the fingers and letting go of it.

3. Counting in one-to-one correspondence with the swing means counting "one" for every complete "round trip" the

bob makes back to the starting point and not for just one-half of the trip, which the children may be inclined to associate with a "swing."

To add interest to the counting, you may wish to have the children start by using the guided counting to determine the duration of various activities in this lesson or you may find that your class needs considerable practice just to learn to count in one-to-one correspondence with the swings.

If you think it appropriate, send home a copy of the letter to the parents which is provided with this unit.

MATERIALS

- 1 pendulum, 5 or 6 feet long
- Letter to Parent
- other objects to be decided by your choice of optional activities

PROCEDURE

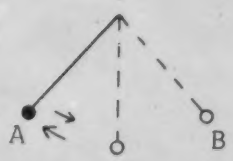
Demonstrate the correct way to start a pendulum. Simply hold the bob a little away from its rest position and let it go -- with no push.

HAS ANYONE THOUGHT OF A WAY THAT WE COULD USE THIS TO GUIDE OUR COUNTING?

There probably will be suggestions that the swings can be counted, but many different ideas about just how the counting should be done.

THERE ARE MANY WAYS WE COULD COUNT, BUT SCIENTISTS USUALLY USE JUST ONE WAY. THEY CALL THE DURATION OF ONE COMPLETE SWING A PERIOD. THEY COUNT PERIODS -- LIKE THIS.

Demonstrate the counting by saying "zero" as you release the bob at A, "one" the first time it gets back to the release point, "two" the next time at A, and so on.



After counting to about six, stop the pendulum and start over again.

THIS TIME I WANT ALL OF YOU TO COUNT OUT LOUD WITH ME.

When you think they are ready, let the children count in unison without your guidance. Then let them count silently.

When you think they have had enough practice, check on each child's ability to count in proper one-to-one correspondence with the swings by having them all count silently for a specific period, and then recording the number of swings. Those with wrong answers should be given more practice.

Optional Activity

When most children are counting satisfactorily, let them use pendulum counting to measure and compare the durations of various activities such as those suggested in earlier lessons. Let various children start the pendulum. Remind them not to push the bob or release it too far away from the rest position.

NOTE: At this time have the children take home the letter to parents that is provided in the Student Manuals.

UNIVERSITY OF *Minnesota*

MINNESOTA SCHOOL MATHEMATICS AND SCIENCE CENTER
720 WASHINGTON AVENUE S.E. · MINNEAPOLIS, MINNESOTA 55414

Office of the Director

Dear Parent,

Your child has been studying a MINNEMAST unit dealing with the concept of time. To prepare him for a better understanding of some of the modern ideas about time, a somewhat unconventional approach has been used. It began by building on the child's intuitive feeling for time by having him learn to compare and measure the durations of various activities. He has been counting while activities are taking place -- and then comparing the number of counts. To do this properly, he needs a device to guide his counting, so that measurements of the time taken by exact repetitions of the same activity will give the same result. In his next lessons, this counting device will be a pendulum made by attaching a few washers or nuts to a strong string suspended from a stick. With this simple device, he can measure durations in terms of the number of swings counted during a given activity. A pendulum is easy to operate and has the added advantage of belonging to a class of counting devices of which our everyday clocks are more complicated examples.

Your child can easily make a pendulum at home. Encourage him to use one to measure and compare durations of many activities. Have him seek answers to such questions as: Which takes more swings of the pendulum, brushing one's teeth or combing one's hair? Making a bed or sweeping the floor? After learning to compare activities according to their durations, your child will study the pendulum itself to discover the relationship between the length of the pendulum and the number of swings it makes in a given interval of time. Then he will study other devices for counting durations of time, using the word "clock" for each device. The flashing directional signals of a car, the rotations of a phonograph record, the human pulse, the ringing of the telephone bell, or even the steady dripping of a leaky faucet -- all of these might serve as possible clocks.

Soon he will begin studying the conventional twelve-hour clock. He will start by watching the second-hand of the clock, discovering that one full revolution of this defines a minute. Then he will learn that the minute hand of the clock is really a counter for the second hand, with the minute hand advancing one small unit on the clock face for each complete revolution of the second hand. He will also start to use the clock to tell time. These various learnings will take place in the next few weeks. Have your child tell you what he is learning about time and clocks as he goes along. Encourage him to use the clock to measure durations of various activities such as preparing for bed, taking a bath, eating a meal, getting dressed and so on -- just as he used the pendulum formerly for shorter durations.

Your interest in your child's experiments will encourage him to increase his understanding of certain aspects of time and reinforce what he is discovering at school.

Cordially,

Teacher

Lesson 23: USING THE PENDULUM TO MEASURE DURATIONS

Here the children are given practice in using the pendulum to measure and compare the durations of activities and to record the results of the measurements.

With the pendulum as a counting guide, the children should measure and compare the durations of a number of events. Use as many activities as are needed to develop these skills. Select some from Lesson 20, or from children's suggestions, or from this list:

1. Printing names on the chalkboard.

2. Allowing a mechanical wind-up toy to run down.

3. Playing a song on a music box or record player.

4. Determining the time for an ice-cube to melt in warm water with stirring and without stirring. (Water should be the same temperature for both experiments.)

5. Moving about the room in various ways such as running, walking, duck-walking.

The worksheet should be completed in the same way as that used in Lesson 21.

The period of the pendulum is used as a duration standard. In the course of this lesson the children should begin to recognize similarities between the measurement of the duration of activities and the measurement of the length of objects. They should be able to note, for example, that the pendulum swing resembles the paper clip as a unit of measurement. The measurements of both length and duration are expressed as intervals. Both durations and lengths may be compared by direct comparisons and indirectly by use of a set of standards. Suggest any similarities that the children do not discover themselves.

MATERIALS

- pendulum suspended from ceiling or other high support, from Lesson 23
- Worksheet 21

PROCEDURE

Have the pendulum set up as before.

CAN ANYONE SUGGEST SOME EVENTS WHOSE DURATIONS WE MIGHT COMPARE BY USING THE PENDULUM?

Select two activities of almost the same duration so that the children will appreciate the usefulness of the pendulum (because of its regularity) in measuring durations that differ only slightly. Let one or two help in deciding which symbols to use to represent the events and in deciding from the data which event had the longer duration.

To help the children realize that there are similarities between the measurement of durations of events and of the lengths of objects, the discussion can be guided by questions such as the following:

HOW ARE WE MEASURING DURATIONS? (By counting pendulum swings.)

HOW DID WE MEASURE LENGTH? (By counting paper clips, inches, etc.)

HOW ARE THESE METHODS SIMILAR? (In each case we counted units.)

WHICH SYMBOLS DID WE USE TO RECORD OUR MEASUREMENT OF LENGTH? (Greater than, less than, appears to be the same as.)

DID WE USE THE SAME SYMBOLS FOR RECORDING OUR MEASUREMENT OF DURATIONS? (Yes.)

HOW DID WE RECORD A MEASUREMENT OF LENGTH THAT CAME OUT BETWEEN TWO REFERENCE LENGTHS? (We placed it between the two lengths.)

DID WE DO SOMETHING LIKE THIS IN MEASURING DURATIONS? (Yes. We found that the duration of an activity was between two durations of counting.)

WHAT ARE SOME WAYS THE MEASUREMENTS OF LENGTH AND OF DURATION ARE ALIKE? (For both, we count units and we use the same symbols.)

Worksheet 21
Unit 12 Name_____

Compare the duration of activities by counting with a pendulum.

1. C_____ < ☐

☐ ..... C_____

2. C_____ < ☐

☐ ..... C_____

3. C_____ < ☐

☐ ..... C_____

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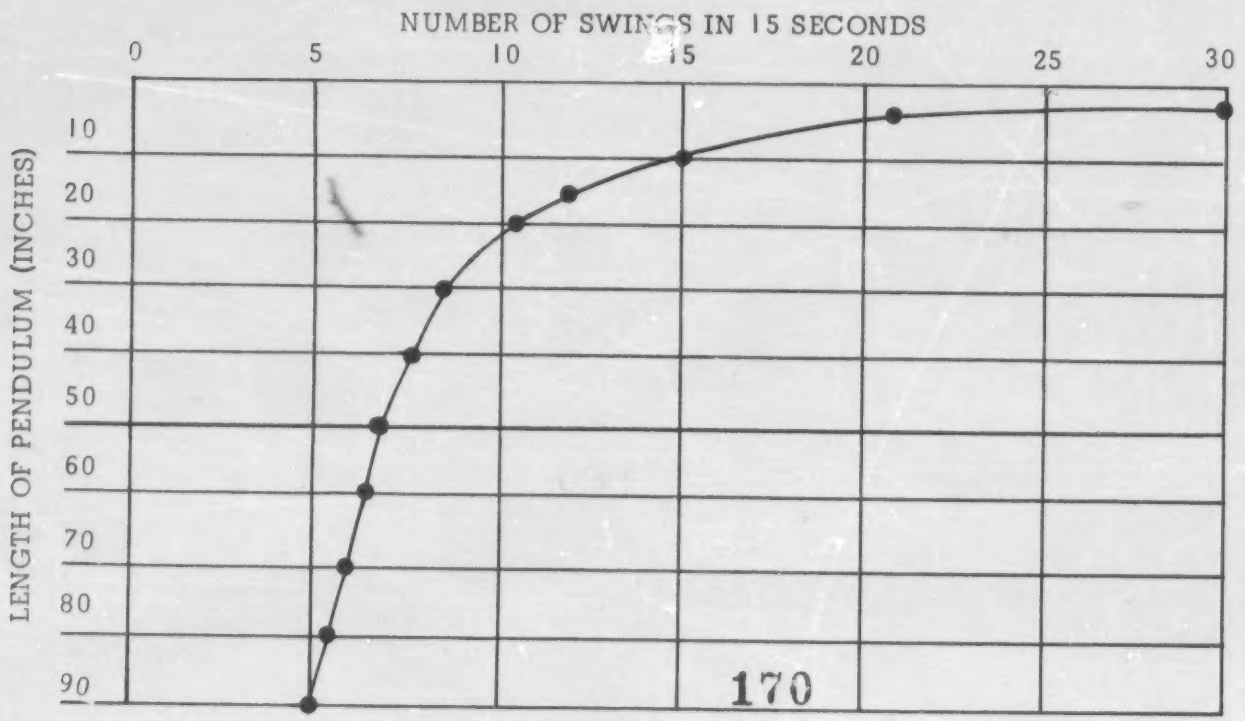
Lesson 24: EXPERIMENTS WITH PENDULUMS

Through experiments, the children begin to develop an understanding of the relationship between the length of a pendulum and the number of swings it makes during a fixed duration. They also begin to make predictions by interpolation.

In this lesson the children will count the number of swings that pendulums of various lengths make during a fixed duration of time. The following table shows the approximate number of swings which pendulums of various lengths will make in fifteen-second time intervals.

Length in inches	2.5	5	10	15	20	30	40	50	60	70	80	90
Number of swings in 15 seconds	30	21	15	12	10.5	8.6	7.5	6.5	6.1	5.6	5.2	5

This table shows that a pendulum which is ten inches long will make fifteen swings in fifteen seconds. It enables you to know beforehand the approximate number of counts that each pendulum will produce. The children are to discover the relationship between the length and the number of swings by experimenting. This graph expresses the above data in a different form.



MATERIALS

- wall clock with sweep second hand
- 8 pendulums of lengths: 72", 50", 20", 12", 8.5", 5.5", 4.5", 3", and a few extra bobs (Mark each bob a different color.)
- Worksheet 22

PROCEDURE

In preparation for this series of activities, devote some time to having the class practice using the motion of the sweep second hand of the clock as an event whose duration they will compare with pendulum swings.

The event will be the motion of the hand from the 12 position to the 3 position and the event will be represented by a simple sketch.

12 position

3 position

event.

Have the class, in unison, say "Start" as the hand reaches the 12 position and "Stop" just as it reaches the 3 position. Repeat several times until you feel that individual children are ready to do it properly.

Divide the class into eight groups and give one of the pendulums to each group. Each group should find a place to suspend its pendulum. The groups with the three longest may need help. Have each group practice releasing the pendulum gently and at a small angle and also practice counting with it.

The groups are to measure the duration of the event of the second hand moving from the 12 position to the 3 position by comparison with their pendulums. The symbol, , will be used on the worksheet to denote the duration of the second hand moving through this angle.

One child watches the clock as the sweep hand approaches the 12. Another child holds the bob out at a small angle ready to release it gently when the clock-watcher says "Start" (at 12). None of the children except the clock-watcher is to watch the

clock, not even the bob-holder. All others are to watch the pendulums and softly count the swings. When the sweep hand passes 3, the clock-watcher calls out "Stop," but the counting children should continue for a few more counts anyway — so that they can observe between what two counts the clock-watcher said "Stop." Then the children record their measurements on the worksheet. They should repeat their counting a number of times until they are consistently getting the same answer each time.

While the class is experimenting, draw a horizontal line about four feet long at the top of the chalkboard. Divide this line, by vertical lines, into equal segments about one and one-half inches long. Above the marks, draw the symbols, C1, C2, C3, etc., to about C32 so they will be clearly visible to the children.

Have each group report to the class its result (in pendulum counts) in determining the duration of the event of the sweep hand going from 12 to 3, while you record the data as a list of numerals on the chalkboard.

WHY DID DIFFERENT GROUPS GET DIFFERENT RESULTS?

ARE THE PENDULUMS ALL THE SAME? (No.)

HOW ARE THEY DIFFERENT? (They differ in length.)

DO YOU SEE ANY RELATIONSHIPS BETWEEN THE LENGTHS AND THE NUMBER OF COUNTS?

Ask one of the groups to bring forward its pendulum and worksheets.

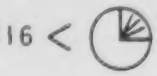
WE WILL TAPE THIS PENDULUM ON THIS NUMBER LINE.

HOW MANY SWINGS DID IT MAKE IN THE EXPERIMENT?

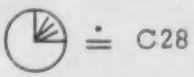
WHERE SHOULD WE HANG IT? (At that count on the number line.)

Have the children slip the paper clip from the support ruler and let them tape the clip just above the line so that the support point is on the line.

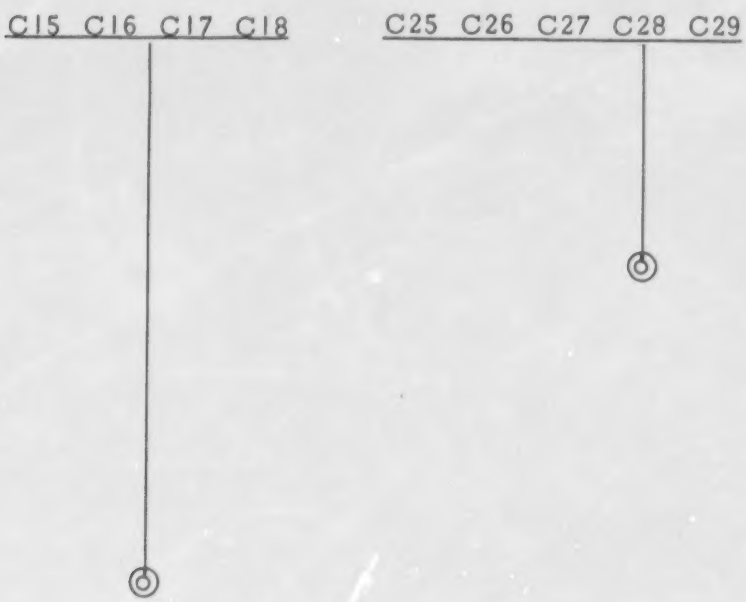
If the measurement were, for example,

C16 <  < C17

then they should tape the pendulum between the marks 16 and 17 on the line, but if the measurement were



they should tape the pendulum on the 28.



Have each group come forward and tape its pendulum on the line. If all the children have made their measurements correctly, the bobs should lie on a smooth curve (a parabola), as illustrated on page 156. However, one or more groups may have made errors in counting, so some of the bobs may look quite out of place.

DO YOU THINK ALL THE PENDULUMS ARE IN THE RIGHT PLACE?

WHAT SHOULD WE DO WITH THIS ONE WHICH SEEMS OUT OF PLACE?

If the children suggest moving it, ask

WHERE SHOULD WE PUT IT?

They may suggest that it be put where it will make the curve smooth — where it will "look right". Whatever they suggest, have them take it down, as well as those others that seem out of place, and check the number of swings again.

After the pendulums have been attached to the board, construct another pendulum (of the same length as any one on the board) and have the children predict how many swings it would make during the same event, a 15-second sweep of the second hand.

CAN ANYONE TELL ME HOW MANY SWINGS THIS PENDULUM WOULD MAKE DURING THE DURATION OF THE CLOCK EVENT?

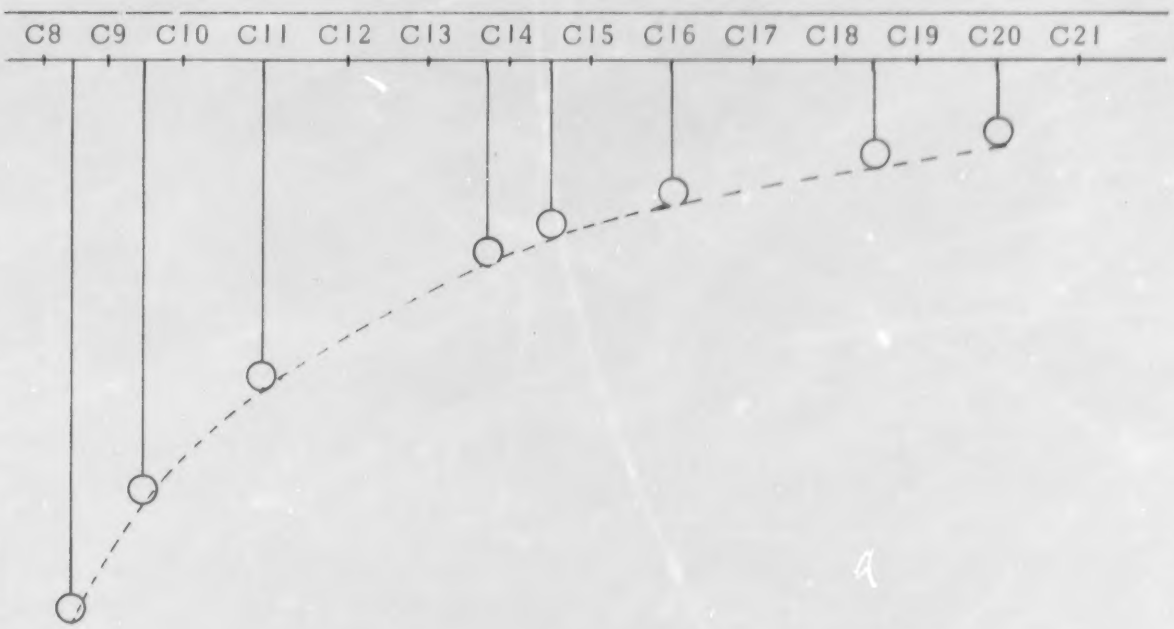
If the children haven't been able to make any guess, help them to see that they might compare the length of the pendulum with those hanging on the board. When they have decided upon a number, have them check it by an actual trial. Have the pendulum attached to the board in its proper place. Repeat with pendulums of lengths not used before (e.g., 60 or 40 inches, etc.)

Next ask the children to describe how they would make a pendulum that would provide a specific number of swings during the event. First choose a number that is associated with the swings of a pendulum already tried, then other numbers (15, 19, etc.)

HOW COULD WE MAKE A PENDULUM WHICH WOULD MAKE ELEVEN SWINGS DURING THE CLOCK EVENT? (By making one the length of the one on the board at eleven.)

After suggestions from the children, each pendulum should be constructed, tested, and then attached to the board with the others.

The children may be encouraged to construct pendulums at home and bring them to hang on the board.

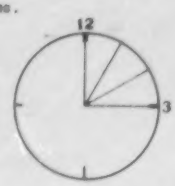


Worksheet 22
Unit 12

Name _____

An experiment with pendulums.

1.

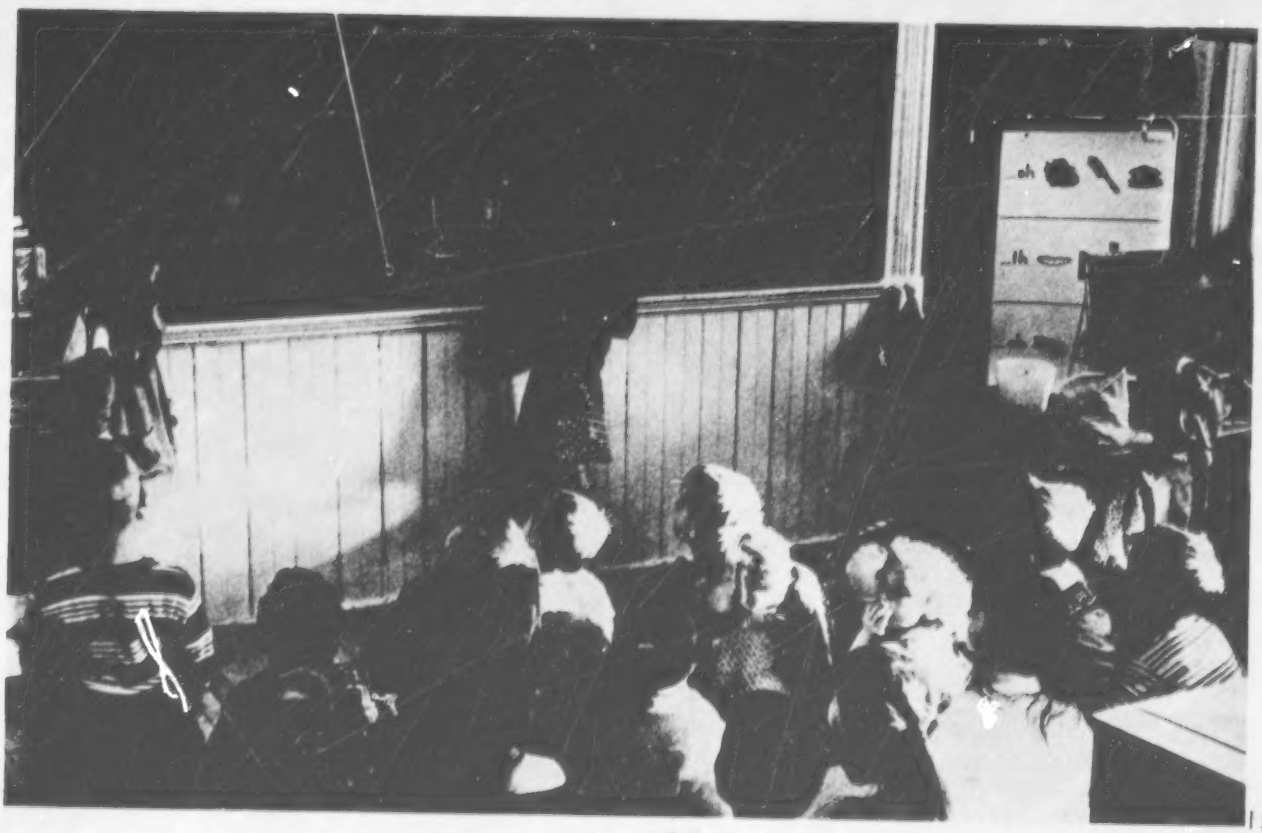
C _____ < 

..... C _____

2.

C _____ <

..... C _____



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Lesson 25: ALL KINDS OF CLOCKS

The activities of this lesson develop the notion of a clock as any device that serves as an aid to counting.

From the variety of possible "clocks" suggested in the list below or those that occur to your class, you should select a few which the children can use to guide their counting in measuring the duration of some event or events. Use only enough unusual "clocks" to give the children the feeling that many things can be used to guide their counting and to aid in measuring durations. By trying others, they will begin to learn of the advantages of the familiar twelve-hour clock.

MATERIALS

- dependent on which "clocks" and events are selected

PROCEDURE

Activity A

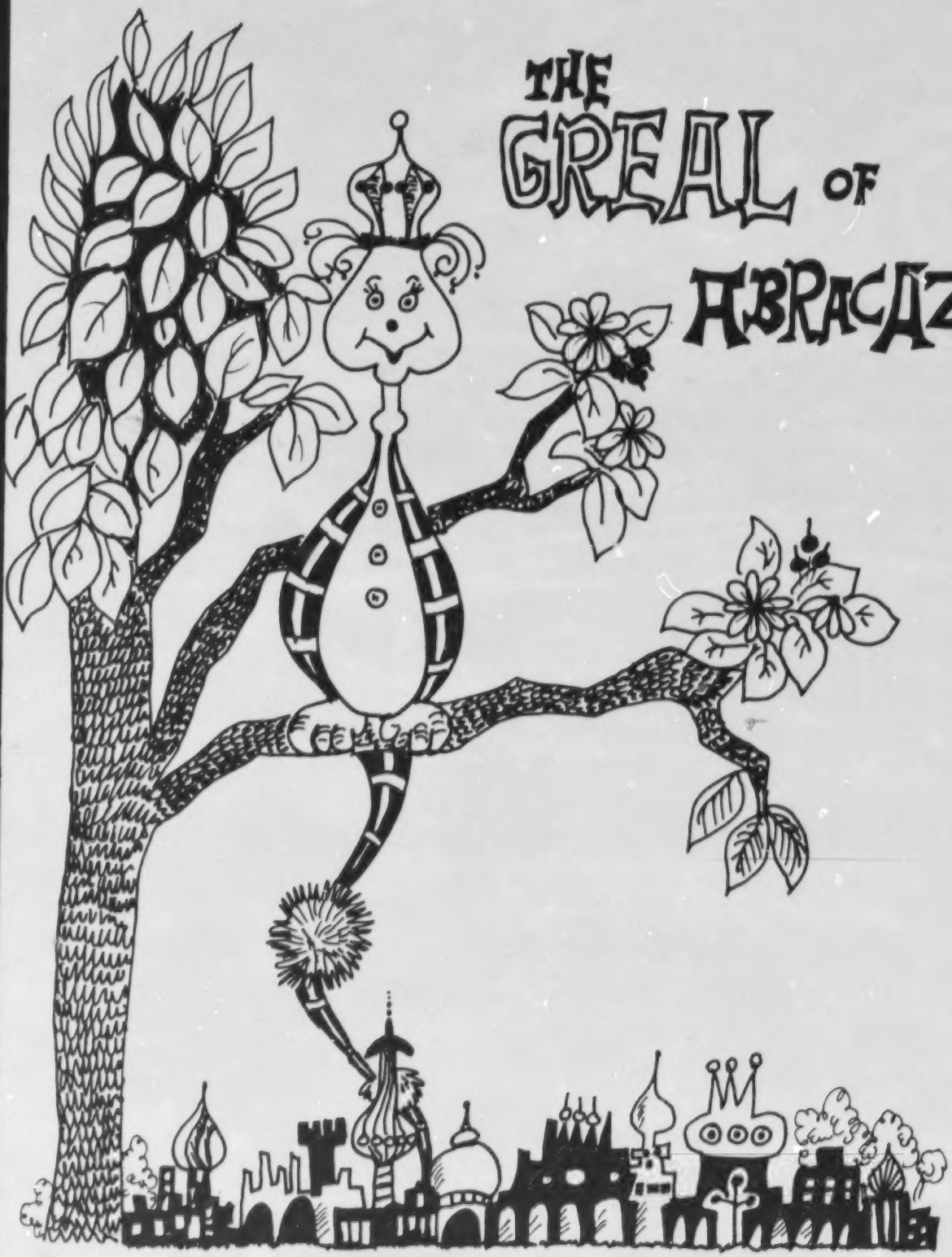
To introduce the idea of strange clocks, read the humorous poem, "The Greal of Abracazand," to the class. After the children enjoy the poem, guide their discussion along these lines:

WHAT DID THE PEOPLE OF ABACRAZAND USE TO MEASURE TIME DURATION? (The Greal)

WHAT DID THEY USE AS THEIR CLOCK? (The wags of the tail or the ringing of the bell, etc.)

WHAT WERE SOME OF THE EVENTS THAT WERE MEASURED BY THE STRANGE CLOCK? (Standing on one's head, skating, etc.)

Use the word "clock" frequently so that the children associate it with the device used to guide the counting.



Poem by Milton K. Snyder

Illustrations by Sonia Forseth

THE GREAT OF ABRACAZAND

In the faraway city of Abracazand
The Greater-Tailed Greal is the clock of the land.
He sits in the highest — the tallest — of trees
And lets his tail wag in the westerly breeze.

The lives of the natives are timed by the Greal
Whose tail is attached to a rope on a wheel
That raises and lowers a sand-filled pail.
Each time that the Greal wags his very long tail,
The pail strikes a bell when it plunks to the ground
And the sound of the bell is heard all around....

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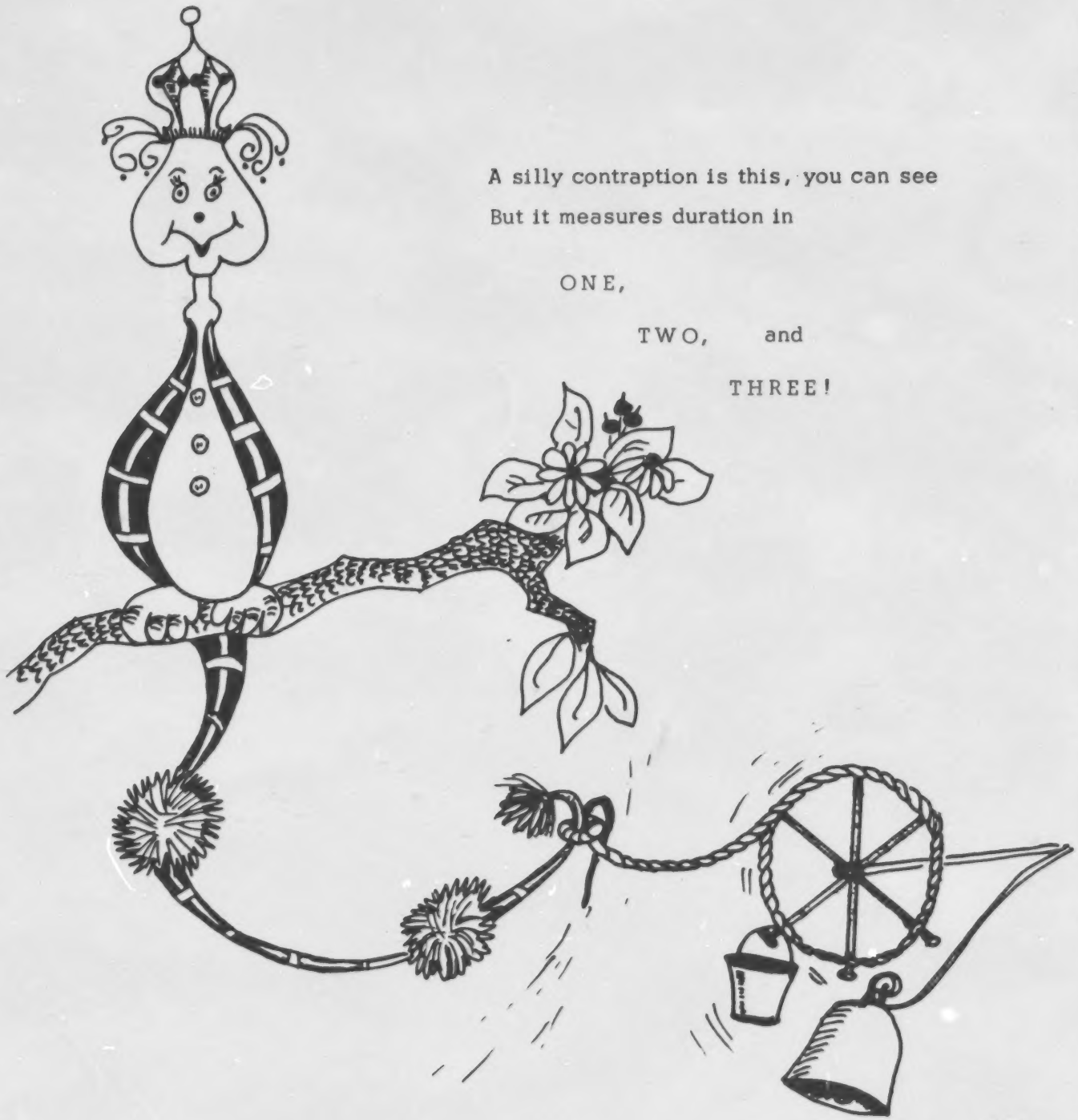


A silly contraption is this, you can see
But it measures duration in

ONE,

TWO, and

THREE!

The natives enjoy many sports of duration;
(In fact, they're the favorite sports of the nation.)
They time everything that they do in their lives
From eating and sleeping and harvesting chives
To playing and working and cooking a meal —
All to the wag of the tail of the Greal.

The record for headstands in Abracazand
Is held by a boy named Peter Q. Rand.
He stood on his head for ten wags of the Greal
Beating the record of old Abner Peel,
Who stood on his head for nine wags, but no more,
Before he came crashing down hard on the floor.

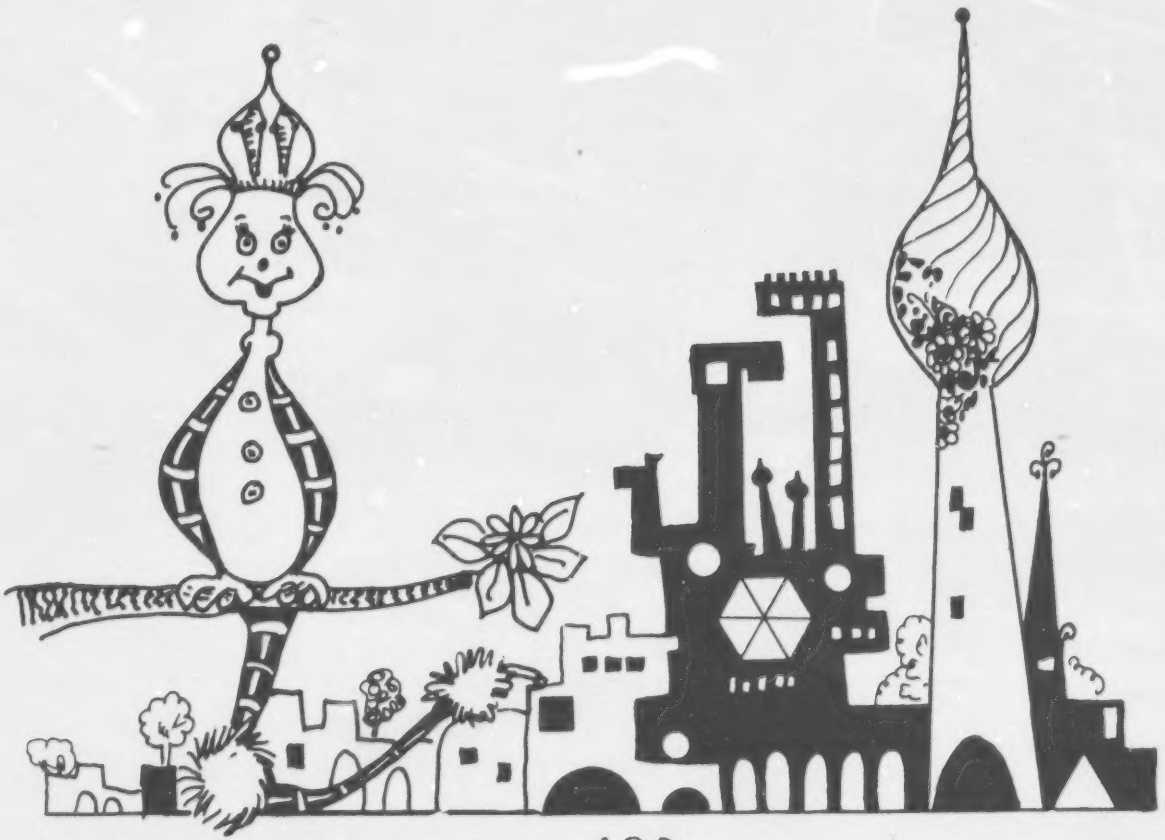
The tail of the Greal had wagged only twice
When Juniper Twittle fell on the ice,
As he skated along on Doohickey Lake.
(Let's not talk about that, for Juniper's sake.)

In the park by Lake Doohickey once every Fall,
A scrumptious picnic's attended by all;
There are races and games and contests galore
And everyone shouts 'til his throat is quite sore.

Everyone likes the potato-sack race
Where anyone's liable to fall on his face,
And the shortest duration for this running sport
Was done by a boy most unusually short.
He covered the course in eight wags of the Greal
By wearing a spring concealed in his heel.

In Abracazand the winters are snowy
It's also the time when the cold winds are blowy.
In one great storm, the snow came down
And covered the city like a fine white gown.
It fell and fell and fell until
Even the Greal could feel the chill;
His tail wagged a thousand times or so
Before the snow began to go.

For a Greal, such weather's a very hard test,
But always it brings out the Greal's very best;
He's able to take every sort of hard knock
For in Abracazand he's the only clock!



Activity B

Have the children suggest devices in the classroom which might be used as "clocks," as the Greal was.

CAN YOU THINK OF ANYTHING THAT WE MIGHT USE AS A CLOCK AS THE PEOPLE OF ABRACAZAND USED THE GREAL'S TAIL?

Either from their suggestions or the list below, select appropriate ones and have the children use them to guide their counting as they measure the duration of various events.

I. Counting aids that might be demonstrated in the classroom:

A. Conventional 12-hour clocks or watches

1. Rotations of the second hand

2. The second hand passing specific numbers

3. Ticking of a clock or watch

B. Rotation of a spot or raised figure on a piece of cardboard placed on a phonograph turntable

C. Water dripping (as from a leaky faucet)

D. Flashing light, such as that used on Christmas trees or a flasher bulb in a flashlight

E. Metronome beats

F. Pulse

G. Breathing (may be irregular)

H. Child bouncing a ball (irregular)

I. Child jumping rope (irregular)

II. Counting aids, outside the classroom, that might be mentioned:

A. Flashing lights

1. Lights on airplane wings

2. Directional signals in automobiles

3. Sweep of a searchlight across the sky

4. Traffic light — cycles (may be irregular)

5. Flashing neon lights on signs

6. Flashing lights on street barricades

7. Flashing lights on radio and TV towers

B. Ringing of a telephone bell

III. Long-term counting aids

A. Clock or Watch

1. Rotation of the minute hand (1 hour)

2. Rotation of the hour hand (12 hours)

B. Day — Noon to noon

C. Week — Sunday to Sunday

D. Year — New Year's Day to New Year's Day

Lesson 26: USING THE STANDARD TWELVE-HOUR CLOCK

In this lesson the children study the conventional twelve-hour clock. The minute is shown to be the duration of the activity of the second hand making one complete revolution. The minute is then found to be the basic unit of duration of which longer duration units are constructed. The minute and hour hands are treated as automatic counters, since, for every complete revolution of the second hand, the minute hand advances one space; and the hour hand advances five spaces (one large space) for one complete revolution of the minute hand. Thus the minute is defined as the duration of one revolution of the second hand, the hour is defined as the duration of one revolution of the minute hand, and a complete 24-hour day is defined as the duration of <u>two</u> revolutions of the hour hand.

The children are led to discover the usefulness of an automatic counter that frees them from the drudgery of continuously watching a counting guide. They find out that, if they know the positions of the clock hands at the start and end of an extended activity, they can compute its duration simply by counting the spaces between the initial and final positions.

MATERIALS

- clock with a clearly visible, sweep second hand. It must have marks for all minutes and should preferably be round.
- demonstration clock face with minute and hour hands
- small cardboard clock faces with movable hands (optional)
- Worksheets 23-29

PROCEDURE

Activity A

Ask the children to watch the second hand of the clock.

COULD WE USE THE SECOND HAND TO GUIDE OUR COUNTING?

HOW SHOULD WE COUNT WITH IT?

They may suggest counting each time it passes a mark (once a second), when it passes a numeral (once every five seconds), or when it completes a trip all the way around an entire revolution (once a minute).

THE DURATION OF THE SECOND HAND MAKING ONE COMPLETE TRIP AROUND IS CALLED A MINUTE. WE WILL COUNT THE NUMBER OF TRIPS AROUND, THE MINUTES.

Use the playing of a record or similar activity as the event whose duration is to be measured.

WE WILL MEASURE THE DURATION OF THIS RECORD BY COUNTING THE TRIPS AROUND OF THE SECOND HAND — THE MINUTES — WHILE I PLAY THE RECORD.

Start the event when the second is at twelve and say "Zero." When the hands gets to twelve again say "One," the next time "Two." Stop the record at "Three."

WHAT WAS THE DURATION OF THE PLAYING OF THE RECORD? (Three counts, three trips around, three minutes.)

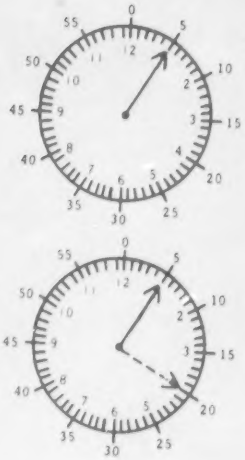
Repeat a few times with events of different durations, each about a whole number of minutes in length.

DID ANYONE NOTICE ANY OTHER CHANGE IN THE HANDS OF THE CLOCK DURING THE EVENT? (The long hand — the minute hand — moved.)

THIS TIME WATCH THE LONG HAND WHILE WE COUNT THE TRIPS AROUND OF THE SECOND HAND — THE MINUTES.

WHAT DID YOU NOTICE? (The long hand moved one space — one mark — for each trip around, each minute.)

Use the large demonstration clock. Tape its hands together. Say that this time you will set the hand on this clock face to show the position of the long hand when the event starts, like this:



Then say that everyone should count the minutes and watch the long hand on the wall clock. Ask, "Where was the long hand when the record stopped?" Then put a mark at that place on the clock face. Use any colored chalk, crayon or tape that is removable.

Lead the children to see that the number of spaces between the hand positions is the same as their number of counts — that the long hand automatically counts and records the number of trips around of the second hand (the number of minutes). If available, use two demonstration clocks. Place them on the chalk ledge. Write "Start" above the clock on the left and "Finish" above the one on the right. One clock can be thought of as a control.

Activity B

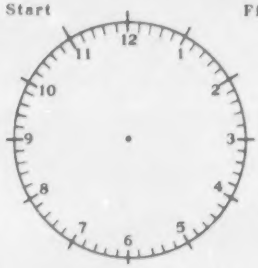
Use Worksheet 23 for determining the duration of longer events. Guide the children to realize that each numeral can be counted in multiples of five when counting minutes. Some may be familiar with the minute markings from their experiences in telling time. Guide them to mark the position of the long hand in the "Start" clock when they begin some activity and on the "Finish" when it ends. The first event should be short — three or four minutes — such as singing a song or taking a few minutes of free time. Longer events could be recess, music period, etc. For each event, the children determine the duration by counting the number of spaces between the two positions of the hand. On Worksheet 24 some durations of possible activities are indicated. These can be used for practice. Since all the events start at zero minutes, the durations are indicated by the "Finish" reading. This may help the children to learn to tell time to the nearest minute.

It should not be expected that all of the children will be able to tell time after these lessons. The activities should be thought of, instead, as experiences on which to build time-telling skills.

Worksheet 23
Unit 12

Name_____

Start

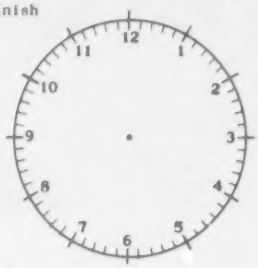


Finish

Duration = _____ Minutes

Start

Finish



Duration = _____ Minutes

Worksheet 24
Unit 12

Name_____

Start

Finish

Duration = 25 Minutes

Start

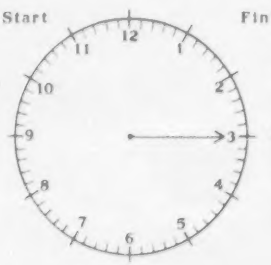
Finish

Duration = 50 Minutes

Worksheet 25
Unit 12

Name_____

Start



Finish

Duration = 20 Minutes

Start

Finish

Duration = 55 Minutes

Worksheet 25 also indicates the duration of imaginary events, but now they start at places on the clock other than 0 minutes. Some children may see that they can determine the durations by some method of subtraction rather than by counting spaces.

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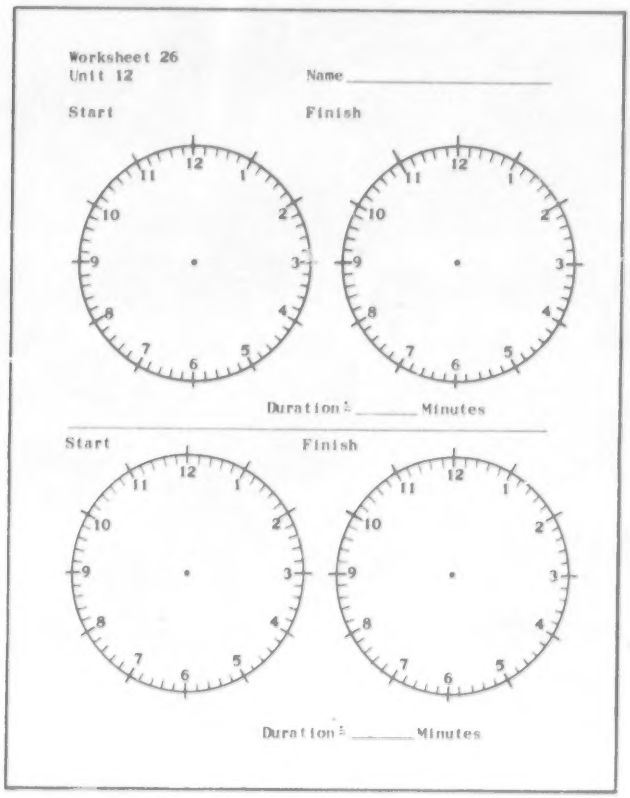
Worksheets 26-29 are blank clock faces which can be used for further practice in determining durations, either those actually observed or imaginary ones that the children or you propose.

You may wish to use the cardboard clocks to supplement worksheets. By having the children keep the short hand under the minute hand, and having them work in pairs, they can use the clocks as they used the drawings to record starts and finishes of events and then determine the durations as on the worksheets.

Worksheets 26 - 29



Lesson 27: MEASURING DURATIONS IN HOURS AND MINUTES

In this lesson the children will be led to see that the hour hand counts the revolutions of the minute hand, just as the minute hand counts the revolutions of the second hand. They will practice measuring durations in hours and, in a few simple cases, in hours and minutes. By considering durations since the hour and minute hands were at twelve, the children will be getting background for telling time in hours and minutes. You should decide how far and how rapidly your class can progress with this material. You may find it best to return to parts of the presentation later in the year. In general, the approach to study of the hour hand will be similar to that of the minute hand.

MATERIALS

- large clock
- small cardboard clocks (optional)
- Worksheets 30 through 34

PROCEDURE

To have the children recognize how the hour hand counts revolutions of the minute hand, have them use Worksheet 30 to record the position of the hour and minute hands, on the hour, several times during a day. (An alarm clock can be used to alert you just before the hour.)

Just before the hour, call the children's attention to the position of the hands and tell them to record the positions when the long hand — minute hand — is at twelve or zero. At the end of the first hour record the positions again. Ask:

HOW MUCH HAS THE MINUTE HAND MOVED SINCE THE OTHER RECORD? (One trip around, sixty minutes.)

HOW MUCH HAS THE SHORT HAND MOVED? (From nine to ten; five small spaces; one large space.)

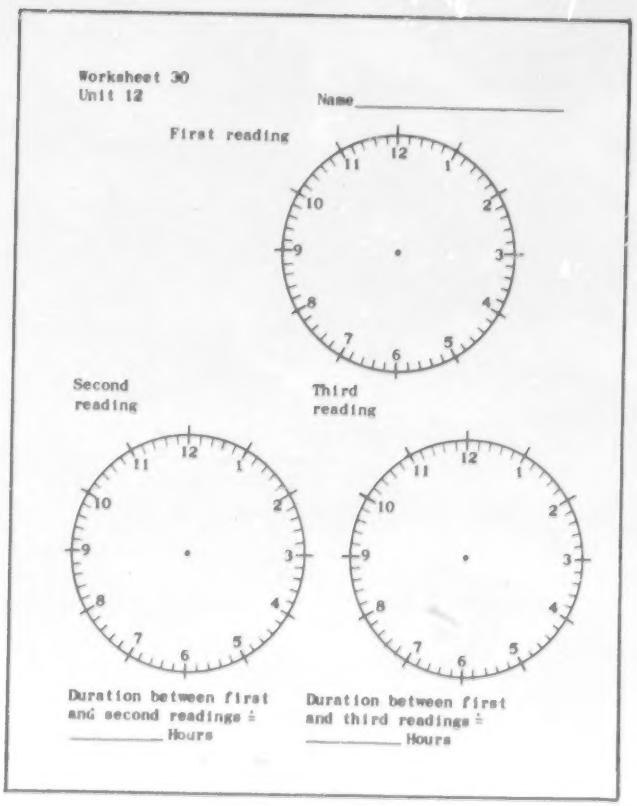
WE SAY THAT ONE TRIP AROUND OF THE MINUTE HAND HAS A DURATION OF ONE HOUR. THE SHORT HAND RECORDS THE TRIPS AROUND OF THE MINUTE HANDS. IT RECORDS THE HOURS.

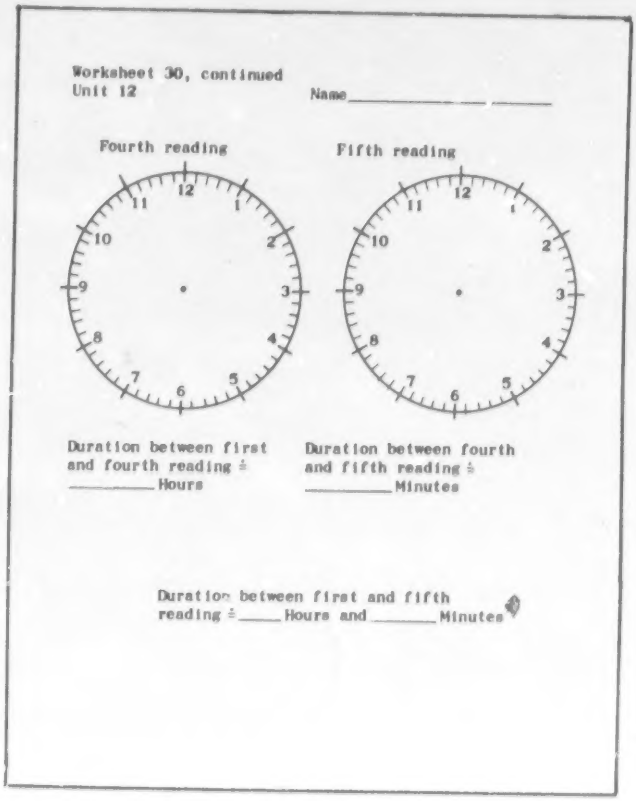
Repeat after the next hour.

Later in the day, skip one of the hourly recordings so that the children can discover that they do not have to watch the clock every hour.

Take the last reading of the day at thirty minutes after the hour. Have the children discuss the durations between the various readings.

WHAT WAS THE DURATION OF OUR WORK BETWEEN THE FIRST READING AND THE SECOND ON THE WORKSHEET? (One hour.)





BETWEEN THE FIRST AND THE THIRD? (Two hours.)

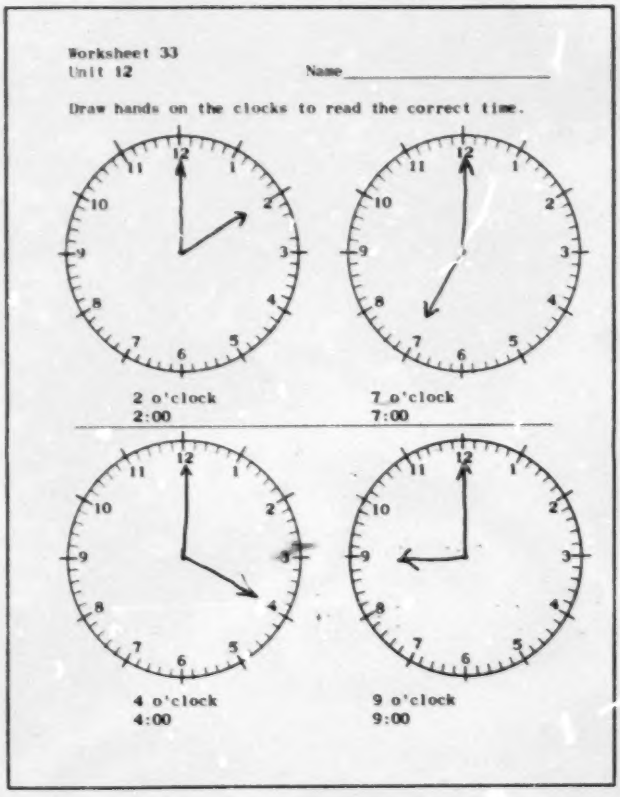
BETWEEN THE FIRST AND THE FOURTH? (Four hours, etc.)

BETWEEN THE FOURTH AND THE FIFTH (LAST)? (Thirty minutes, one-half hour.)

THINK CAREFULLY NOW. WHAT WAS THE DURATION BETWEEN THE FIRST AND THE FIFTH? (Four hours and thirty minutes or four and a half-hour.)

Worksheets 31 and 32, which are duplicates of Worksheet 23, can be used for further drill of this sort, either with actual observations of the clock or with imaginary data. A few children may want to handle more complicated minute situations.

Worksheets 33 and 34 give the children practice in drawing hands on clocks and then reading each indicated time.


Worksheet 33
Unit 12
Name _____

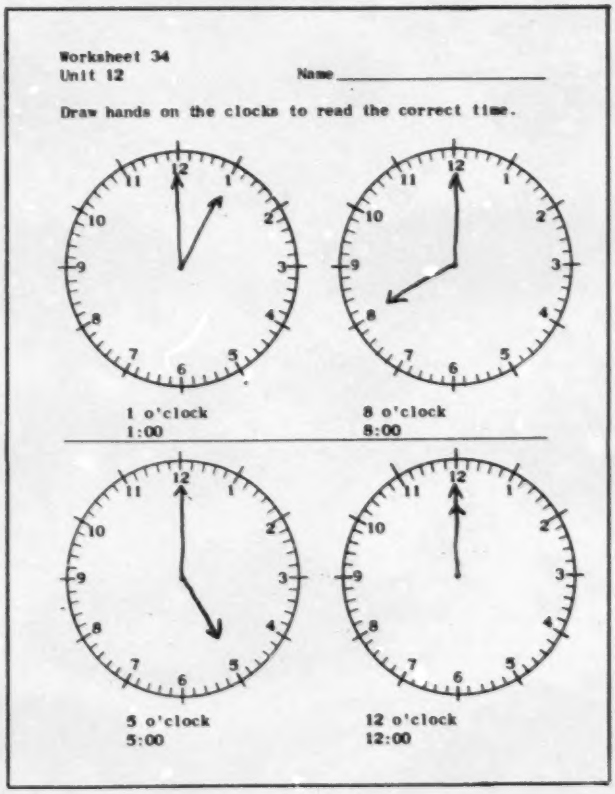
Draw hands on the clocks to read the correct time.

2 o'clock
2:00

7 o'clock
7:00

4 o'clock
4:00

9 o'clock
9:00


Worksheet 34
Unit 12
Name _____

Draw hands on the clocks to read the correct time.

1 o'clock
1:00

8 o'clock
8:00

5 o'clock
5:00

12 o'clock
12:00

Lesson 28: TELLING TIME

This lesson provides activities which associate telling time with the duration of events extending from noon or midnight. It is intended only as a suggestion of one way to present clock reading as an extension of the study of time duration. To put the various units of time duration in perspective, a time line — a modified number line — is introduced and gradually extended from part of a day to weeks, months, years, etc.

Use Worksheets 35-38. On 35, the durations of several events are recorded. In each case, the event is assumed to have started at a time when the minute hand and the hour hand were both at twelve or zero. Point out that the durations were all from noon (in the afternoon). Also point out that when such a duration is two hours, we say the time is "two o'clock" and we write it "2:00." Worksheet 36 associates the position of the hands of the clock with a time line (modified number line) on which the durations since noon are indicated and the recording of the times shown. Worksheet 37 extends the clock reading to the morning and then to the whole day, introducing "A.M." and "P.M." notations. Worksheet 38 shows how the time line can be extended to include days, months, years. It is unlikely that you will find it advisable to cover all of these aspects of the time line now. Some extension can be done as you continue to review time-telling during the balance of the year. Other extensions will be included in the second grade work.

A suggestion of a story which might be used to motivate the worksheets is provided. You probably will want to let the children use cardboard clock faces along with their worksheets.

MATERIALS

- Worksheets 35-38
- cardboard clocks

PROCEDURE

Present the worksheets by means of the story, "Tommy and the Clock," or in any way that seems best to you. However you do it, emphasize the relation between durations since noon (or midnight) and clock readings.

TOMMY AND THE CLOCK

Tommy's father came home from a long business trip on Saturday morning. Tommy was so glad to see him! And Mother celebrated by preparing a delicious brunch, with coffee cake and scrambled eggs, and all the good things that Tommy and his father especially liked. It was nearly noon when they all sat down to eat.

"Well, son, what have you been studying in school these last two weeks?" Daddy asked.

"Time durations, Daddy. A time duration is how long it takes to do something. First we counted the durations with a pendulum, but now we're learning about clocks like this one." Tommy pointed to the clock on the wall above the kitchen sink.

"Why, that's great!" Daddy said. "Can you tell me what time it is now?"

Tommy saw that both the long hand and the short hand were pointing straight up. "Yes," he said proudly, "it's twelve o'clock."

Tommy helped himself to another piece of coffee cake, then looked at the clock again, and said, "There are some things I'm still trying to figure out, though. I promised Greg I'd play catch with him at two o'clock, and now I'm wondering if there is any connection between two hours' duration and two on the clock."

"That's a very interesting question," Daddy said, "but I think you can work out the answer for yourself if I draw some pictures of clock faces for you."

Daddy reached in his pocket, pulled out a pencil and started to draw a clock face on his paper napkin. Tommy laughed, because Daddy so often started to scribble or draw on the napkins. Tommy went and found some larger paper for Daddy.

"Thank you, Tommy," Daddy said, "We can get more clock faces on this paper." Then he drew some clocks and told Tommy to figure out the durations between where the hands were in one picture of a clock and where they were in another picture right beside it.

Tommy figured out the durations and he was absolutely right each time.

"Well," Daddy said happily, "it looks as though we will have another expert time-teller in the family in no time."

Tommy felt very proud.

Have the children turn to Worksheet 35. Explain that these are problems just like those Tommy's father gave him. Tell them that they should look first at the clock on the left (the start position) and then at the clock on the right (the finish position) and figure out the duration for each of the three problems. When they have done this, you can conduct a discussion — still carrying the story line — like this:

TOMMY'S FATHER ASKED HIM SOME QUESTIONS ABOUT THESE CLOCK FACES. I WANT YOU TO ANSWER THE SAME QUESTIONS.

Worksheet 35
Unit 12

Name_____

Start

Finish

Duration = 2 Hours

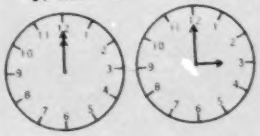
Time 2 o'clock P.M

Duration = 3 Hours

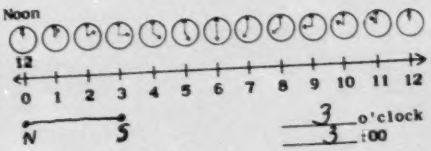
Time 3 o'clock P.M.

Worksheet 36
Unit 12

1. Noon School Day Ends

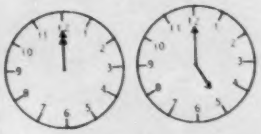


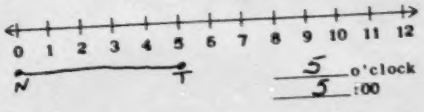
Noon



3 o'clock
3 :00

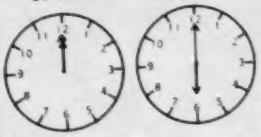
2. Noon T.V. Program

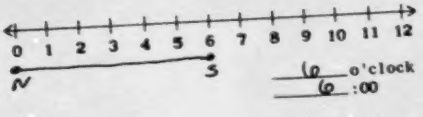




5 o'clock
5 :00

3. Noon Supper





6 o'clock
6 :00

Name_____

FATHER ASKED: WHAT WAS ALIKE ABOUT ALL THREE OF THESE CLOCK EVENTS? WHAT DO YOU THINK TOMMY ANSWERED? (They all started when the hands were together at the top of the clock.)

THEN FATHER SAID: WHEN THE HANDS ARE TOGETHER LIKE THAT IN THE MIDDLE OF THE DAY, WE SAY IT IS NOON. WHEN DID ALL OF THESE EVENTS START? (At noon.)

IN PROBLEM 1, WHAT WAS THE DURATION SINCE NOON? (Two hours.)

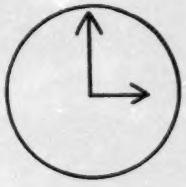
WHEN THE DURATION SINCE NOON IS TWO HOURS, WHAT DO WE SAY THE TIME IS? CAN YOU GUESS? (Two o'clock.)

THAT'S RIGHT. THE TIME WOULD BE TWO O'CLOCK. WE CAN ALSO WRITE IT AS "2:00."

THEN TOMMY'S FATHER MADE ANOTHER SET OF PROBLEMS FOR HIM — JUST LIKE OUR WORKSHEET 36.

HE ASKED TOMMY: WHEN YOU FINISHED SCHOOL YESTERDAY, DO YOU REMEMBER HOW THE HANDS WERE ON THE CLOCK?

TOMMY DREW A PICTURE SOMETHING LIKE THIS:



LOOK AT PROBLEM ONE ON WORKSHEET 36. WHAT DURATION DOES IT SHOW? (Three hours.)

THEN, WHAT TIME WAS IT WHEN SCHOOL ENDED? (Three o'clock, or 3:00.)

LET'S LOOK AT THE NUMBER LINE ON THE SHEET. WHAT DO YOU NOTICE? (There are clock faces above it.)

WE CALL SUCH A LINE A TIME LINE. LET'S PUT A DOT UNDER THE TIME LINE AT NOON AND LABEL IT N . AND A DOT UNDER THE TIME LINE FOR THE END OF SCHOOL AND LABEL IT S .

HOW CAN WE SHOW THE DURATION SINCE NOON FOR THE END OF SCHOOL? (Draw line from N to S.)

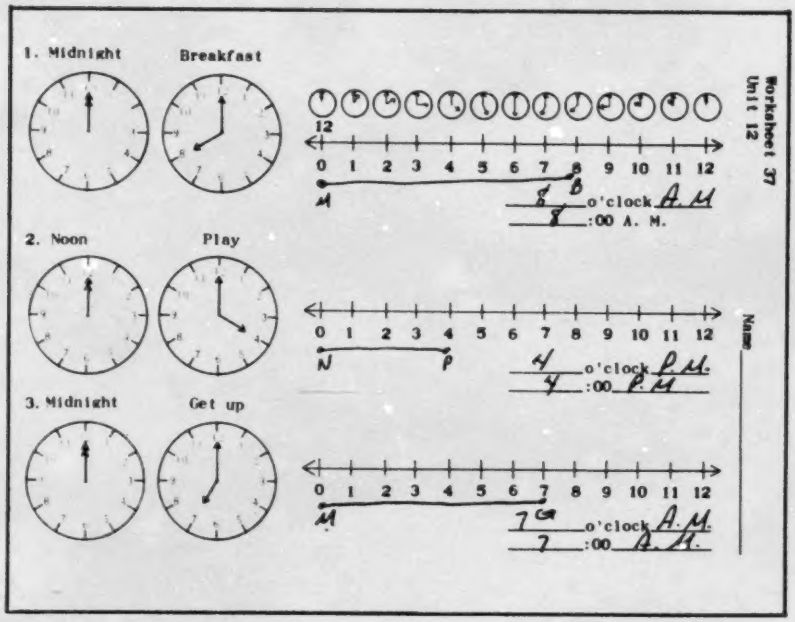
WHAT WAS THE DURATION SINCE NOON FOR THE END OF SCHOOL? (Three hours.)

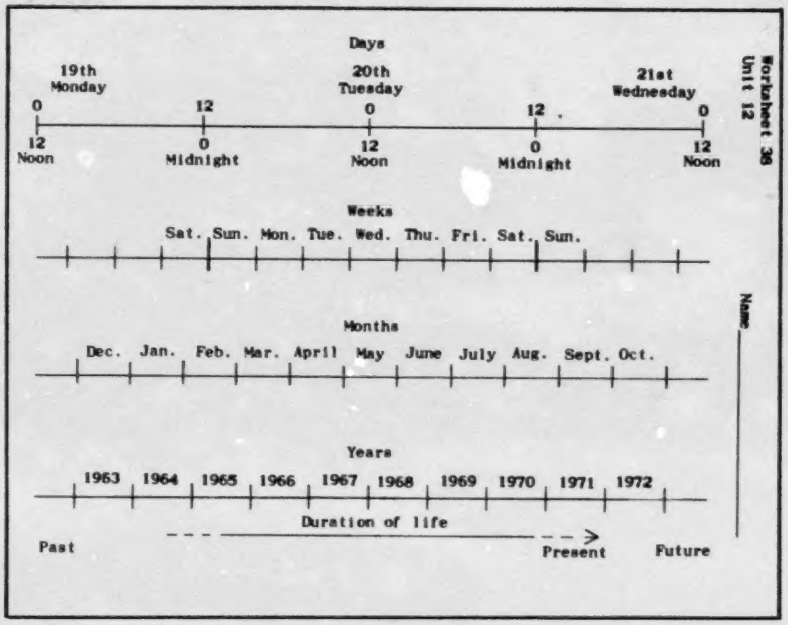
WHAT TIME DID SCHOOL END? (Three o'clock, 3:00.)

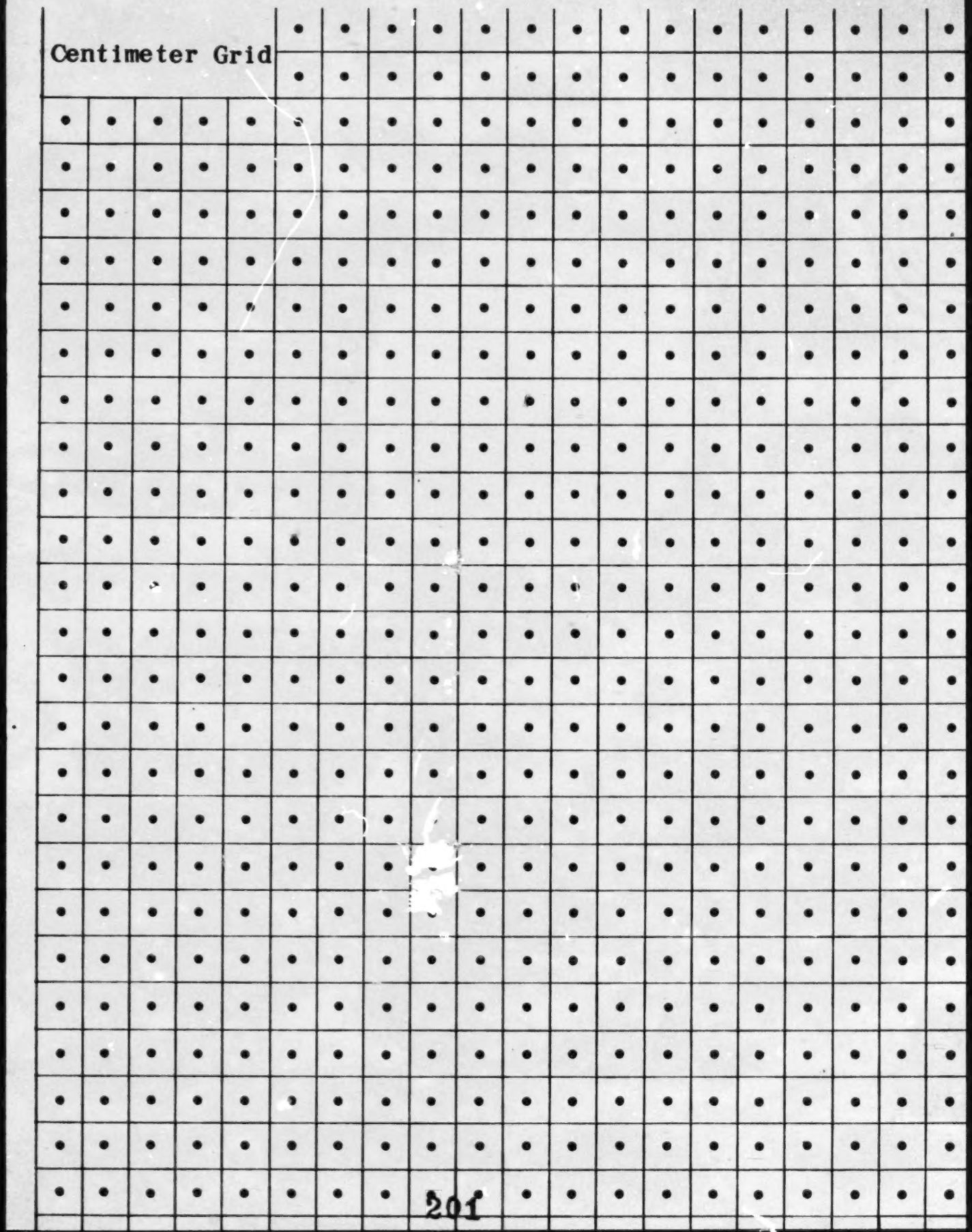
Continue by saying that next Tommy figured out the time for his favorite TV program, his supper and for going to bed.

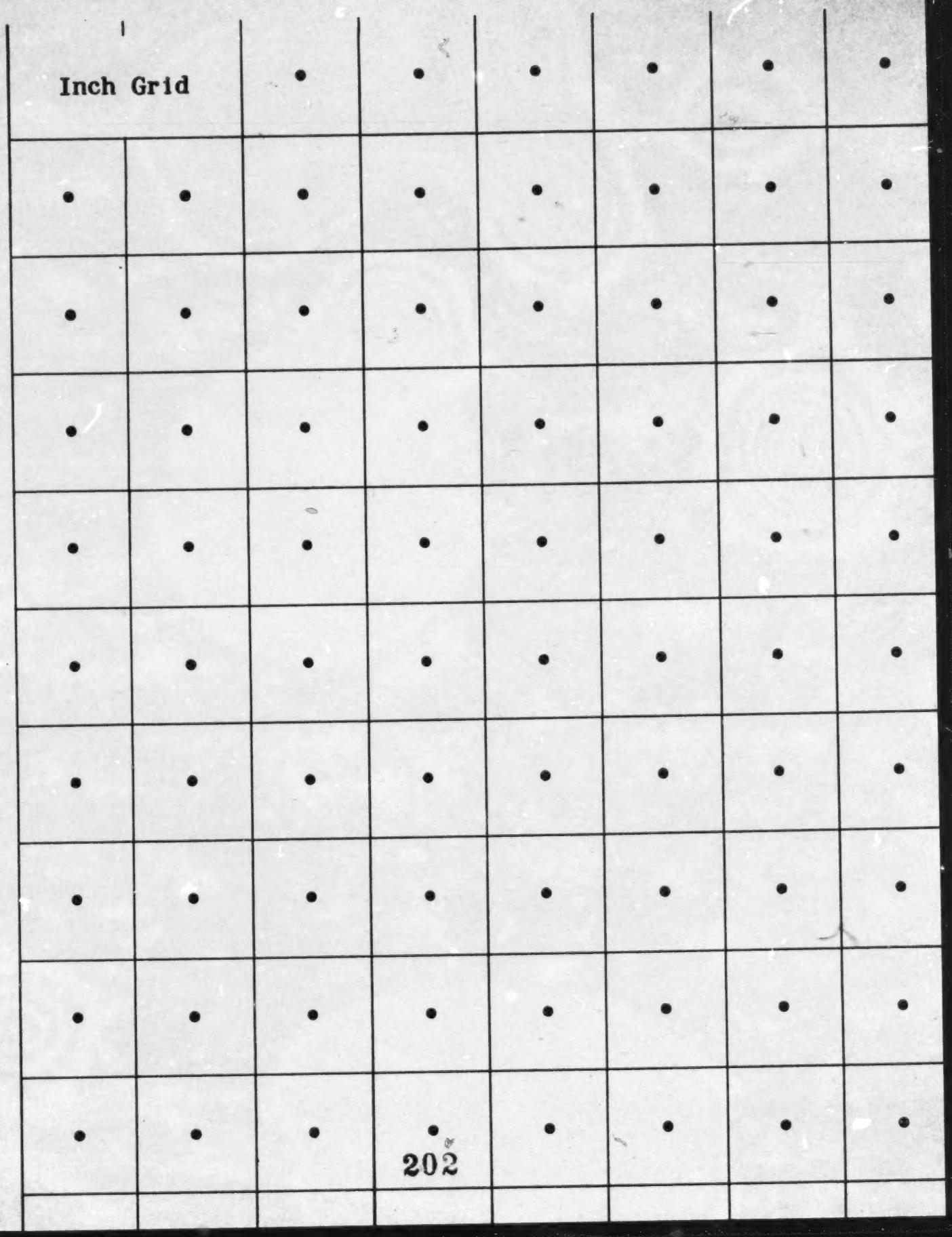
And that several days later, Tommy and his father talked about time again. This time, they extended the time line before noon and found that durations from midnight (when the hands are together in the middle of the night) are in the morning and are labeled A.M. (Latin for "before noon" is "ante meridian.") The durations after noon are marked P.M. (Latin for "after noon" is "post meridian"). (Worksheet 37.)

Later, Tommy found that the time line can be extended to include days, weeks, months, years, etc. (Worksheet 38.)

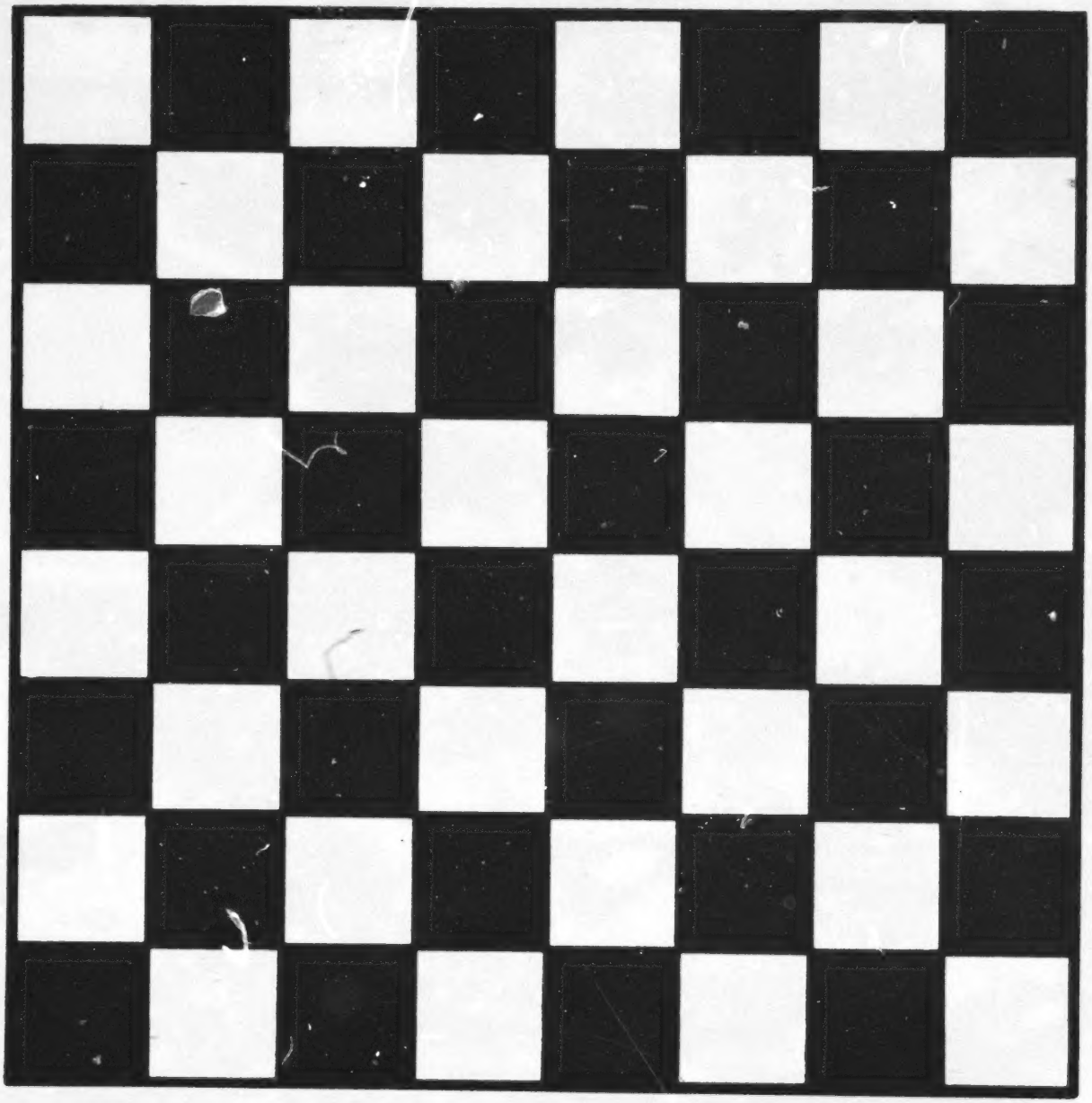

Worksheet 37
Unit 12
Name
1. Midnight
Breakfast
12
0 1 2 3 4 5 6 7 8 9 10 11 12
M
8 o'clock A.M.
8 :00 A. M.
2. Noon
Play
0 1 2 3 4 5 6 7 8 9 10 11 12
N P
4 o'clock P.M.
4 :00 P.M.
3. Midnight
Get up
0 1 2 3 4 5 6 7 8 9 10 11 12
M
7 o'clock A.M.
7 :00 A.M.


Worksheet 38
Unit 12
Name
Days
19th Monday
20th Tuesday
21st Wednesday
0 12 0 12 0
12 Noon
0 Midnight
12 Noon
0 Midnight
12 Noon
Weeks
Sat. Sun. Mon. Tue. Wed. Thu. Fri. Sat. Sun.
Months
Dec. Jan. Feb. Mar. April May June July Aug. Sept. Oct.
Years
1963 1964 1965 1966 1967 1968 1969 1970 1971 1972
Duration of life
Past
Present
Future

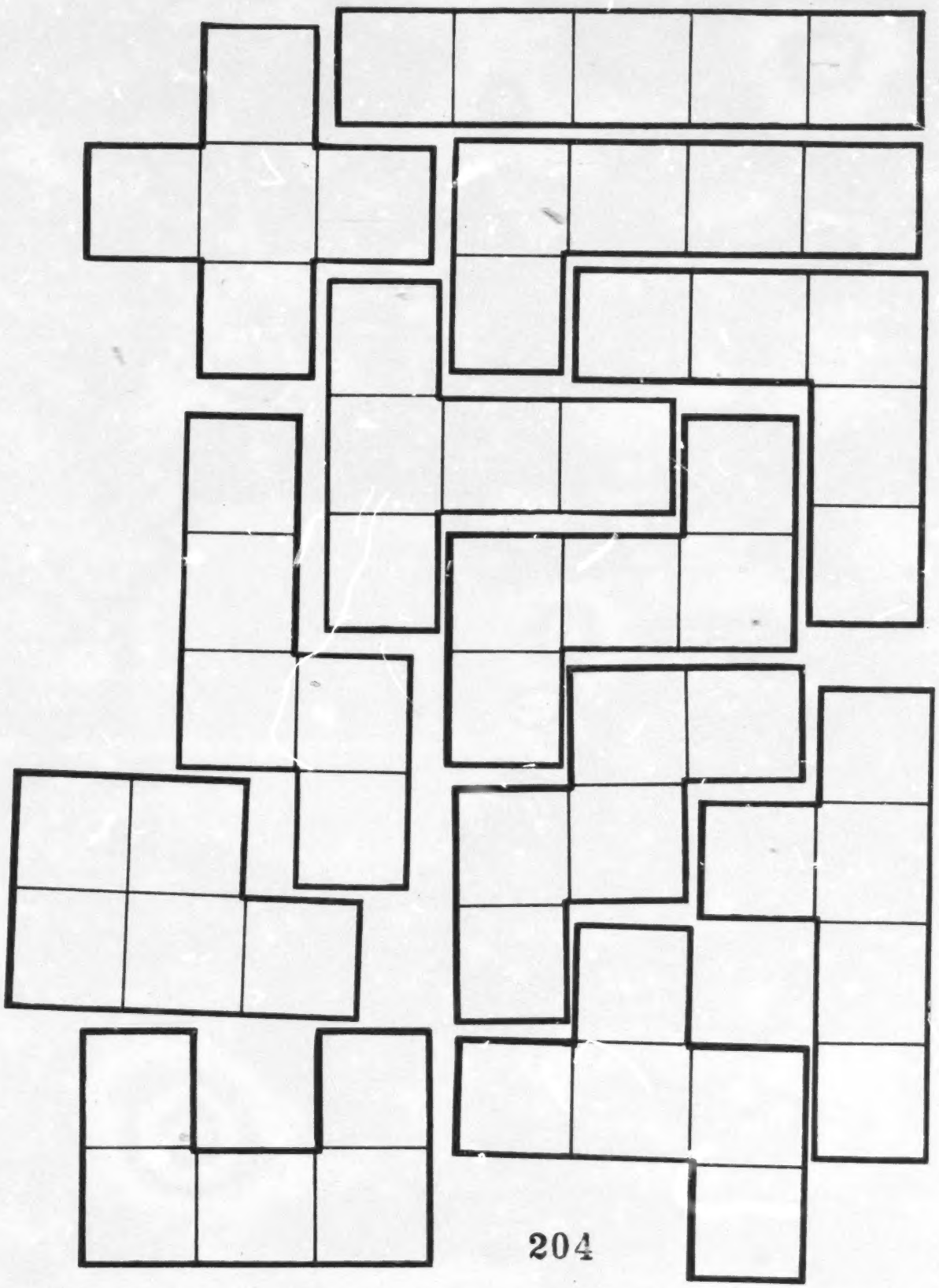

Centimeter Grid
201


1 Inch Grid

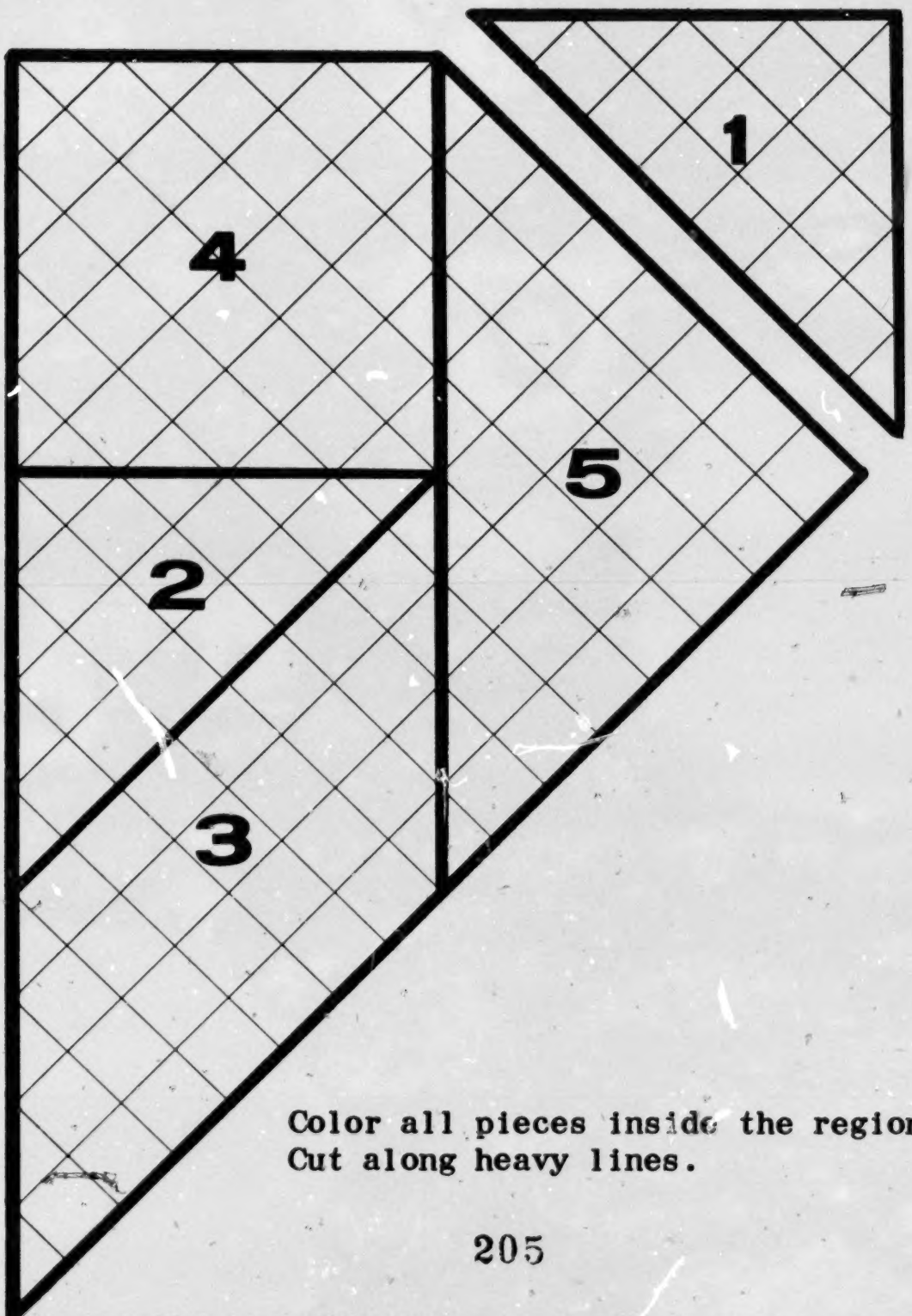
Pentomino Game Board



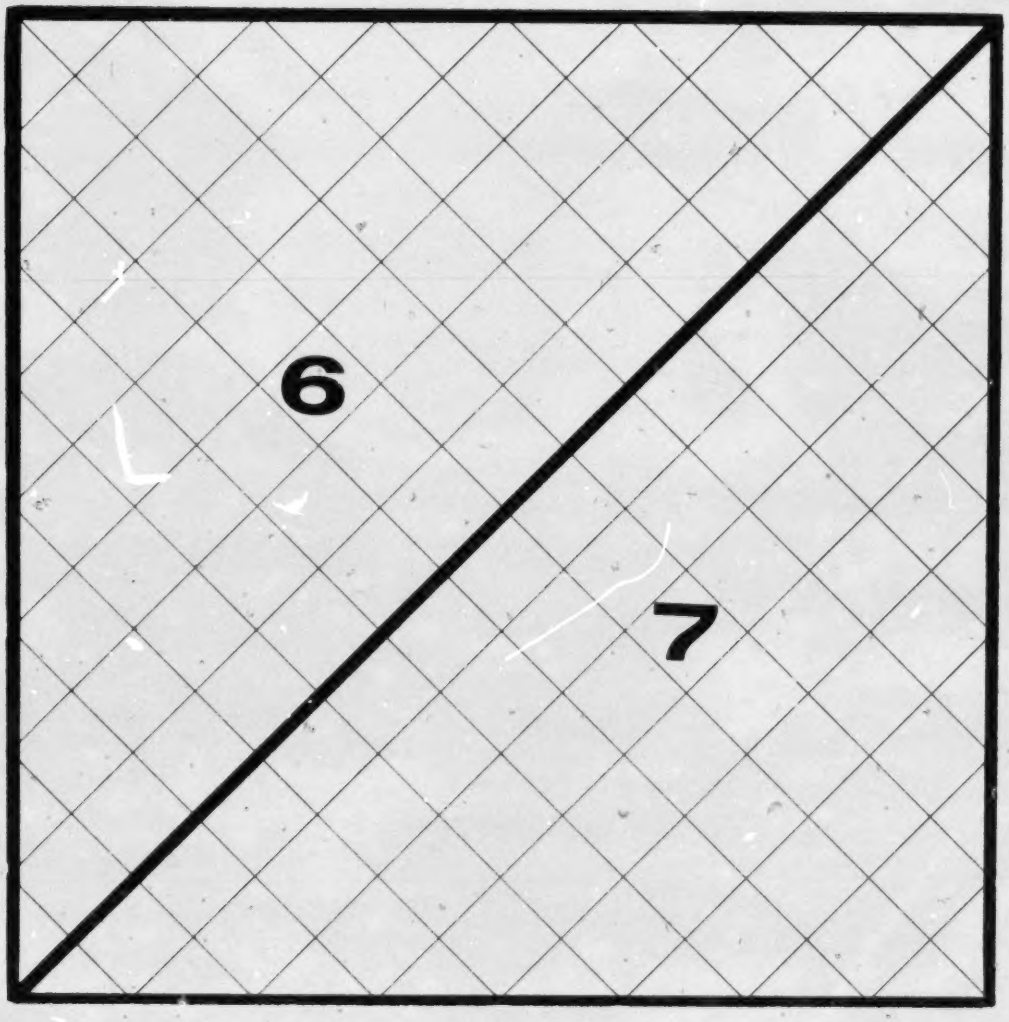
Pentomino Game Pieces



Tangram Puzzle



Color all pieces inside the regions.
Cut along heavy lines.

Tangram Puzzle

Color all pieces inside the regions.
Cut along heavy lines.